SubOptimal Continuous-Curvature Path Planning for Non-Holonomic Robots

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Résumé - *Depuis 1986, la planification de chemins pour robot mobile de type voiture a donné lieu à de nombreux travaux. Cependant, la quasi totalité d'entre eux génèrent des chemins constitués de segments de droite reliant tangentiellement des arcs de cercle de courbure maximale. Ces chemins sont en effet les plus courts, que le robot se déplace uniquement en marche avant (cela a été démontré par Dubins en 1957) ou qu'il fasse des manoeuvres (Reeds et Shepp, 1990).*

Pourtant, depuis 1990 justement, plusieurs travaux portant sur le contrôle des robots mobiles ont mis en avant l'importance de la continuité de la courbure pour obtenir des chemins dont le suivi soit précis. Or, les chemins couramment utilisés ne vérifient pas cette propriété. D'autres types de chemins ont été proposés mais la pertinence du choix de ces chemins, par rapport au modèle de robot considéré, n'a pas été discutée.

Le travail de ma thèse est le premier, à ma connaissance, à avoir intégré la contrainte de continuité de la courbure dans le problème de planification. Il a de plus étudié les caractéristiques de ce nouveau problème, en terme d'existence de solution et d'existence et de nature des solutions optimales. Enfin, dans le cas sans manoeuvre, il a proposé un planificateur générant des solutions sous-optimales (proches de l'optimal) dont la complexité et le temps de calcul sont équivalents à ceux du planificateur de Dubins. Des expérimentations ont permis de montrer que les chemins fournis par ce planificateur sont suivis, avec une loi de commande de type Kanayama, avec une précision dix fois supérieure à celle obtenue dans le cas des chemins de Dubins. De plus, un séjour post-doctoral a permis de montrer que ces chemins sont aussi particulièrement intéressants pour des robots plus manoeuvrables (sans borne sur la courbure), lorsque des vitesses importantes sont considérées.

Mots clé - robotique mobile, planification de chemin, sous-optimalité, continuité de la courbure.

Abstract

Since 1986, numerous works have been focusing on path planning for car-like mobile robot. However, most of these works generate paths made of line segments tangentially connecting circular arcs of maximum curvature. These are indeed paths of minimum length, whether the robot goes only forward (this has been proved by Dubins in 1957) or both forward and backward (Reeds et Shepp, 1990).

Nevertheless, several works about mobile robots' control insisted on the importance of the continuity of the curvature, in order to obtain paths that can be followed precisely. While the paths usually used do not verify this property, other types of paths were presented but the relevance of the choice of these paths, w.r.t. the robot's model, was not discussed.

The work presented in my thesis is the first, to the best of my knowledge, which adds the continuity of the curvature as a new constraint for the path planning problem. The characteristics of the resulting problem, in term of existence of solution as well as existence and nature of optimal solution, are presented. At last, a method is proposed to plan forward-only suboptimal paths, with a complexity and a computation time similar to those of Dubins' planner. Experimental results showed that the tracking of these Suboptimal Continuous-Curvature paths (or *SCC paths*), using a Kanayama control law, is ten times better than the tracking of Dubins' paths. At last, a post-doctoral research proved that these SCC paths can also be used for more manoeuvrable robots (*i.e.* robots with an unbounded curvature), as soon as high velocities are considered.

1 Introduction

This paper focuses on path planning for a car-like robot: given two positions of this robot, we search a path connecting these positions and avoiding collision with a set of obstacles. The path considers only the geometrical aspects of the movement (no time dimension), but needs to respect two classical kinematics constraints: the direction of motion must remain parallel to the main axis of the robot at each point, and the turning radius of the robot is lower bounded.

Numerous methods have been proposed to solve this problem, e.g. [13, 7, 16, 25, 23], using paths made of circular arcs of minimum radius tangentially connected by line segments. The optimality (in length) of these paths has been proved by Dubins in the forward-only case [5], and by Reeds and Shepp for a robot doing backup manoeuvres [18]. The drawback of these paths is the discontinuity of their curvature profile, which makes them difficult to follow by a real robot.

To reduce this disadvantage, paths with a continuous curvature profile can be computed, these paths having polynomial coordinates [10, 17] or polynomial curvature [9, 4]. But few of these generators consider mobile robots with a bounded curvature, as for example [15, 24, 17], and none of them do take into account obstacle avoidance. Moreover, these works do not present any theoretical results concerning the existence of solution paths or the nature of optimal solutions: in fact, the problem considered is rarely clearly stated. At last, no comparison between these methods have been proposed, and the improvement of the tracking has never been measured.

Some theoretical results have been obtained by Boissonnat, Cerezo and Leblond [1] and developed by Kostov and Kostova [11, 12] for manoeuvrable robots, *i.e.* when the curvature is continuous but unbounded and the derivative of the curvature is bounded. Boissonnat, Cerezo and Leblond showed the existence of solutions, characterized the optimal paths and proved that these paths are generally made of an infinity of pieces (and thus cannot be used). Then Kostov and Kostova presented a set of sub-optimal paths to solve this problem.

This paper first state formally the planning problem considered (§ 2) then prove the characteristics of this problem in term of existence of solutions, existence and nature of the optimal paths (§ 3). A path planning method is proposed in the forward-only case (§ 4). At last, experimental results obtained with this method, as well as application for more manoeuvrable robots, are presented (§ 5).

2 Statement of the Problem

In order to state the problem we consider in this work, we will present the model of our robot and the paths it can follow.

2.1 The Car-like Robot

Our robot $\mathcal{A}$ is similar to a car-like vehicle moving on a planar environment. Its body is a rectangle supported by four wheels: the two rear wheels' axle is fixed to $\mathcal{A}$'s body and the two front wheels are directional. A position of this robot is given by a configuration (x, y, θ, κ), where (x, y) are the coordinates of a reference point R of the body, θ is the orientation of the body (*i.e.* the angle between the x axis and the main axis of $\mathcal{A}$) and κ is the instantaneous curvature of R's curve and represents the orientation of the front wheels (cf. Fig. 1). The idea of adding the instantaneous curvature to the classical configurations comes from [1], its advantage will be described in the next section.

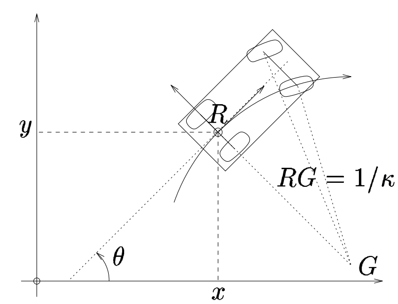

Figure 1: a car-like vehicle.

The robot $\mathcal{A}$ moves on a planar workspace $\mathcal{W}$, which is represented by a compact (*i.e.* closed and bounded) set of $\mathbb{R}^2$. This workspace is cluttered with a set of obstacles $\mathcal{B}_j$, $j \in \{1, \ldots, b\}$, represented by polygonal regions. The body of $\mathcal{A}$ must avoid **contact and collision** with these regions.

The motion of $\mathcal{A}$ is also limited by two classical constraints, as the four wheels of $\mathcal{A}$ should roll without sliding. Considering the rear wheels, whose axle is fixed to $\mathcal{A}$'s body, it implies that the movement of R remains at each point parallel to the main axis of the robot, *i.e.*:

$$\dot{x} \sin\theta - \dot{y} \cos\theta = 0 \tag{1}$$

On another hand, the orientation between the directing wheels and $\mathcal{A}$'s main axis is bounded, which implies that the turning radius RG is lower bounded, or that the curvature of R's curve is upper bounded:

$$|\kappa| \leq \kappa_{\max} \tag{2}$$

At last, the orientation of the directing wheels can change with a limited speed only, and thus the derivative of the curvature of R's curve remains also bounded:

$$|\dot{\kappa}| \leq \sigma_{\max} \quad (3)$$

2.2 Feasible Paths

A path is a continuous set of positions of $\mathcal{A}$. It can be represented by a continuous curve of the configuration space $\mathcal{C} \subset \mathcal{W} \times \mathcal{S}^1 \times [-\kappa_{\max}, \kappa_{\max}]$. As a consequence, **its curvature profile** (the fourth coordinate of the configurations) **is continuous**. It is *feasible* if and only if it respects the constraints (1), (2) and (3), and is of finite length.

A feasible path can be represented by its projection on $\mathcal{W}$, i.e. by the curve R follows along this path: the orientation θ along this path is deduced using the constraint (1), its existence being ensured by the constraint (2). This representation is usually used for graphic display.

A feasible path is *smooth* if and only if its projection on $\mathcal{W}$ is C^2: along such a path, the robot moves always in the same direction (forward or backward), without back-up manoeuvres. A smooth path can also be represented by its starting configuration $q(0)$, its length l and its curvature profile $\kappa : [0, l] \longrightarrow [-\kappa_{\max}, \kappa_{\max}]$, (with $|\dot{\kappa}| \leq \sigma_{\max}$).

2.3 Planning Problem

A path Π is a mapping from $\mathbb{R}$ to $\mathcal{C}$, giving a configuration $q(s)$ for each $s \in [0, l]$, where l is the length of Π. Given a start configuration $q_s = (x_s, y_s, \theta_s, \kappa_s)$ and a goal one $q_g = (x_g, y_g, \theta_g, \kappa_g)$, such a path is a solution to our problem if and only if it links q_s to q_g and is feasible and collision-free, *i.e.*:

1. End conditions: $q(0) = q_s$ and $q(l) = q_g$;

2. Π is feasible, and therefore its curvature profile is a continuous function *almost everywhere*[1] $\kappa : [0, l] \longrightarrow [-\kappa_{\max}, \kappa_{\max}]$, such that $|\dot{\kappa}| \leq \sigma_{\max}$;

3. Π is collision-free:

   $$\forall j \in \{1, \ldots, b\},\ \forall s \in [0, l],\ \mathcal{A}(q(s)) \cap \mathcal{B}_j = \emptyset$$

   where $\mathcal{A}(q(s))$ denotes the region of $\mathcal{W}$ occupied by $\mathcal{A}$ when in the configuration $q(s)$.

[1] A property is verified **almost everywhere** if and only if it is verified everywhere except for a finite number of values.

The planner presented in this paper only generates smooth paths (*i.e.* forward-only paths) between configurations whose curvature is null. The generalization to forward and backward paths, as well as paths between configurations with non-zero curvature, will be considered this summer.

3 Properties of the Problem

Before searching a solution to this problem, it is interesting to know whether such a solution exists. This question can be answered by proving the controllability of the robot in the considered problem. On another hand, it is also interesting to know if optimal solutions (i.e. paths with a minimum length) do exist, and to characterize these solutions.

3.1 Controllability

We have proved two results, which are equivalent to the controllability properties for the problems considered by Dubins and by Reeds and Shepp (i.e. the classical problems, without curvature continuity).

The robot is *controllable* if and only if there exists a feasible and smooth path linking any two configurations. The controllability of the robot has been proved analytically [21, § 5.3]. It means that there always exists a solution to the problem considered, as soon as there is no obstacle in the workspace.

On another hand, the robot is *small-time controllable* when it does backup manoeuvres: for each configuration q and each neighbourhood $\mathcal{V}$ of q, there exists a second (smaller) neighbourhood of q where every configuration can be reached from q along a feasible path included in $\mathcal{V}$ [21, § 4.1]. It means that the kinematic constraints do not limit the existence of solutions: a feasible and collision-free path exists between two configurations if and only if a collision-free path exists between these configurations [13, Prop. 5].

3.2 Optimal Paths

Filippov's existence theorem for Lagrange and Bolza problems of optimal control, as stated in [3, th. 5.1.ii], can be applied to our problem in order to prove the existence of a solution path of minimum length as long as a solution path exists [21, § 4.2].

The results obtained by Boissonnat, Cerezo and Leblond [1] for manoeuvrable robots can be translated to our problem. It implies that optimal paths are made of line segments, circular arcs of maximum curvature and

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pieces of clothoid[2] of maximum derivative of the curvature. However, these optimal paths usually contain an infinity of pieces, and thus cannot be used in practice [21, § 4.2].

3.3 Considered Paths

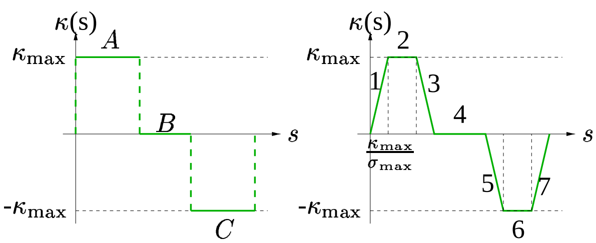

Figure 2: curvature profiles of Dubins' and SCC paths.

Thus, to solve our problem, we chose to use simpler paths than the optimal ones: these path are made of at most 9 pieces, of the same kind as the pieces of the optimal paths (line segments, circular arcs and pieces of clothoid), and are called SCC paths (for SubOptimal Continuous-Curvature paths). These paths are very similar to Dubins' paths, but have a continuous curvature profile: the discontinuities of the Dubins' paths are replaced by pieces of clothoid (cf. Fig. 2). The sub-optimality of these paths have been proved analytically [21, § 6].

4 Path Planning Method

The path planning is performed using a classical method: a fast and simple planner, called *local planner*, is associated with a higher level method to obtain the *global planner*. The local planner does not take obstacles into account, it only search the shortest (feasible and smooth) path linking two configurations, while the higher level method deals with the collision avoidance. We will mainly consider the local planner, which is our contribution to this method. A detailed presentation of this planning method can be found in IROS'97 proceedings [20], or in the French thesis [19].

The local planner is similar to Dubins' planner: it searches at most six paths, the circular arcs of Dubins' paths being replaced by *continuous-curvature turns*, made of three pieces: in Fig. 2, A is replaced by the turn made of the pieces 1, 2 and 3, and C is replaced by the turn 5–6–7. In order to apply Dubins' method, we need to find the set of the configurations that can be reached from a given configuration with a continuous-curvature turn of various deflection (i.e. variation of the orientation). It means that, for a given configuration q, we will search the set described by the final configuration of a continuous-curvature turn starting at q, when the deflection of this turn changes from 0 to 2π.

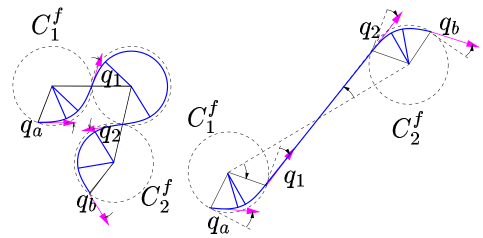

Figure 3: two examples of local planning.

We proved in [20, § 4] that, with a few simplifications, the configurations of this set are placed on a circle of fixed radius R_T, with their orientation doing a constant angle γ with the tangent to the circle (and a zero curvature). Local planning is then done with a method similar to Dubins' one, except that the circles considered have different positions and bigger radiuses (*cf.* Fig. 3).

5 Applications

5.1 Car-Like Robots

In this section, we will mainly compare Dubins' local planning with ours, w.r.t. the complexity, the length of the paths generated and the quality of the tracking. We will also present some experimental results obtained with the global planner we implemented. Details can be found in IROS'98 proceedings [21, § 6].

Complexity of the Computation Dubins' and our local planner have the same algorithm, the formulas in our case being a little more complex. They have therefore the same complexity, and equivalent computing time: the computation of a SCC path is between 1.5 to 2 times longer than the computation of a Dubins' path (over a million tests). Including collision checking to local planning increases similarly the time needed in both case: the computation time ratio is nearly the same with or without collision checking.

Length of the Paths Sub-optimality of the SCC paths has been proved analytically with a large bound [21, § 6.2]. However, experimental comparison between the

[2] A clothoid is a curve whose curvature is a linear function of its arc length.

length of Dubins' paths and the length of the SCC paths intuitively shows a smaller bound: more than 82% of the SCC paths are less than 10% longer than the corresponding Dubins' path, and the standard deviation of the length ratio is less than .2.

Quality of the Tracking Examples of tracking have been simulated, using a Kanayama's law as described in [8]. The maximum distance between the planned path and the followed path in more than ten times smaller when using SCC paths. In "wide turns" paths, the turns (circular arcs in Dubins' case, or continuous-curvature turns in our case) are followed by long line segments: in Dubins' case, the control method can come back to the planned path before arriving to the next turn. In "zigzags" paths, the turns are adjacents: the tracking errors of the turns add one to the other. With a velocity of 1 meter per second, the SCC paths are followed to the centimeter, while the distance to Dubins' paths is about 35 cm. in "wide turns" and 1.5 meter in "zigzags". With a velocity of 3 meter per second, the control method comes to a blocking situation with Dubins' zigzags. In our case, the tracking errors always remain smaller than 20 cm., and are equivalent for both types of paths.

The SCC path planner has been used with a reactive fuzzy controller [6], to control one of the experimental vehicle (a Ligier) of the SHARP project.

5.2 Manoeuvrable Robots

Paths generated by this planner can be used for suboptimal trajectory planning for more manoeuvrable robots, *i.e.* robots with an unbounded turning radius. The optimal trajectories for these robots indeed contain line segments, pieces of clothoids [14] and circular arcs (the radius of these arcs correspond to bounds of velocities). An example of application of this planner to suboptimal continuous-curvature trajectory planning for manoeuvrable robots is given in [22].

6 Conclusion and Future Works

This paper describes a new path planning problem, adding two curvature constraints (continuity and bound on its derivative) to the classical kinematic constraints of a car-like robot. Our contributions are the characterization of the problem, w.r.t. the existence of solution and the optimality of these solutions, the definition of a planning method in the forward-only case and experimental comparisons between Dubins' local planner (usually used) and the continuous-curvature local planner. The complexity of this one is equivalent to the complexity of Dubins' one, and its computation time is less than twice Dubins' one. On another hand, Dubins' paths are more than ten times harder to track than continuous-curvature paths. The continuous-curvature local planner is thus more efficient than Dubins' one.

Future works will explore two main directions: improvement of the forward-only planning (planning between configurations with non-zero curvature, determination of the type of the shortest path as a generalization of the work of Bui et al. [2]) and study of the case with manoeuvres (definition of an optimality criteria, nature of the optimal paths, positions of changes of motion's direction).

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