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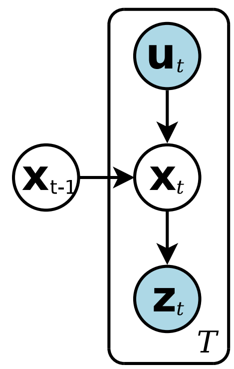
Information Processing in Robotics

Solution Sheet 3

Topic: Online estimation: application to localization and mapping

Exercise 1: Kalman filter

In this exercise, we will investigate in more details the equations of the Kalman filter. To do that, we will rely on some relations for Gaussian distributions.
Assuming:



- $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda})$,
- $p(\boldsymbol{y} \mid \boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}, \boldsymbol{L})$,

we have:

- $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}|\boldsymbol{A}\boldsymbol{\mu} + \boldsymbol{b}, \boldsymbol{L} + \boldsymbol{A}\boldsymbol{\Lambda}\boldsymbol{A}^{\mathrm{T}})$,
- $p(\boldsymbol{x} \mid \boldsymbol{y}) = \mathcal{N}\left(\boldsymbol{x}|(\boldsymbol{\Lambda}^{-1} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{L}^{-1}\boldsymbol{A})^{-1}\left\{\boldsymbol{A}^{\mathrm{T}}\boldsymbol{L}^{-1}(\boldsymbol{y} - \boldsymbol{b}) + \boldsymbol{\Lambda}^{-1}\boldsymbol{\mu}\right\}, (\boldsymbol{\Lambda}^{-1} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{L}^{-1}\boldsymbol{A})^{-1}\right)$.

(a) $P(\boldsymbol{x}_t|\boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{0:t})$ can be computed using the first formula with: $\boldsymbol{A} = \boldsymbol{F}$, $\boldsymbol{b} = \boldsymbol{B}\boldsymbol{u}_t$, $\boldsymbol{L} = \boldsymbol{Q}$, $\boldsymbol{\mu} = \hat{\boldsymbol{x}}_{t-1|t-1}$, and $\boldsymbol{\Lambda} = \boldsymbol{P}_{t-1|t-1}$. This yields:

- $\hat{\boldsymbol{x}}_{t|t-1} \leftarrow \boldsymbol{F}\hat{\boldsymbol{x}}_{t-1|t-1} + \boldsymbol{B}\boldsymbol{u}_t$,
- $\boldsymbol{P}_{t|t-1} \leftarrow \boldsymbol{F}\boldsymbol{P}_{t-1|t-1}\boldsymbol{F}^{\mathrm{T}} + \boldsymbol{Q}$.

(b) $P(\boldsymbol{x}_t|\boldsymbol{z}_{0:t}, \boldsymbol{u}_{0:t})$ can be computed using the second formula with now: $\boldsymbol{A} = \boldsymbol{H}$, $\boldsymbol{b} = \boldsymbol{0}$, $\boldsymbol{L} = \boldsymbol{R}$, $\boldsymbol{\mu} = \hat{\boldsymbol{x}}_{t|t-1}$, and $\boldsymbol{\Lambda} = \boldsymbol{P}_{t|t-1}$. This yields:

- $\hat{\boldsymbol{x}}_{t|t} = (\boldsymbol{P}_{t|t-1}^{-1} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\left\{\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}(\boldsymbol{z}_t - 0) + \boldsymbol{P}_{t|t-1}^{-1}\hat{\boldsymbol{x}}_{t|t-1}\right\}$,
- $\boldsymbol{P}_{t|t} = (\boldsymbol{P}_{t|t-1}^{-1} + \boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}$.

(c) If we introduce $\boldsymbol{K}_t = \boldsymbol{P}_{t|t-1}\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{H}\boldsymbol{P}_{t|t-1}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R})^{-1}$ we can use the matrix identities to fall back on the Kalman filter algorithm.

(d) See code.

(e) In the code, it requires having an expression for both the transition model and its Jacobian which cannot be generic anymore.