Non-euclidean virtual reality

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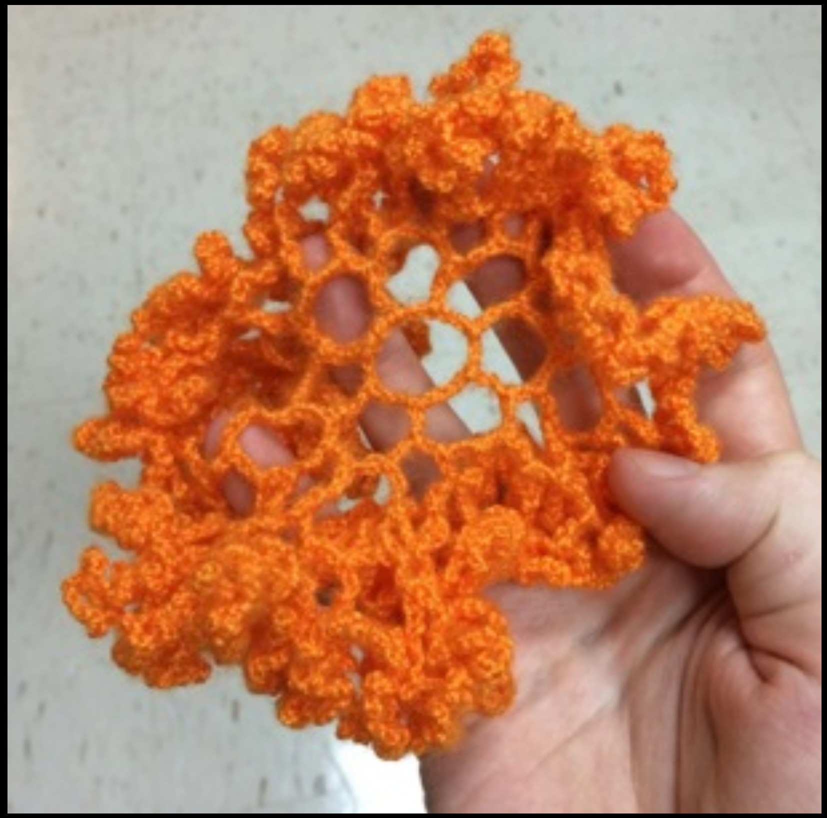


These are all pictures from the “outside”…

What does it look like from the inside?

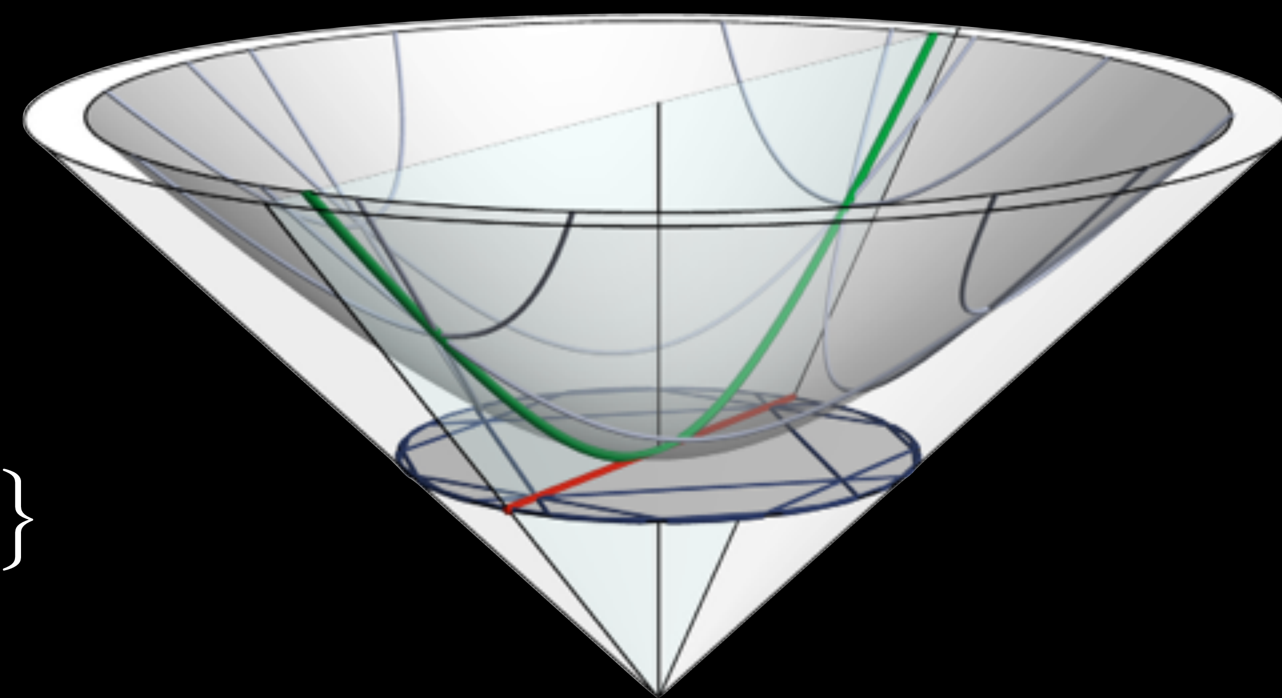
Light rays should follow geodesics

Creating a non-euclidean virtual world:

1. A model of the space
2. A way to draw points in that space on the screen
3. A way to move around that space
4. A set of landmarks to help the viewer navigate that space

Modeling $\mathbb{H}^3$

The Hyperboloid Model



Hyperboloid in Minkowski $\mathbb{E}^{3,1}$

$$\{\{x_1, x_2, x_3, w\} \in \mathbb{E}^{3,1} | \\ x_1^2 + x_2^2 + x_3^2 = w^2 - 1, w > 0\}$$

The Klein Model

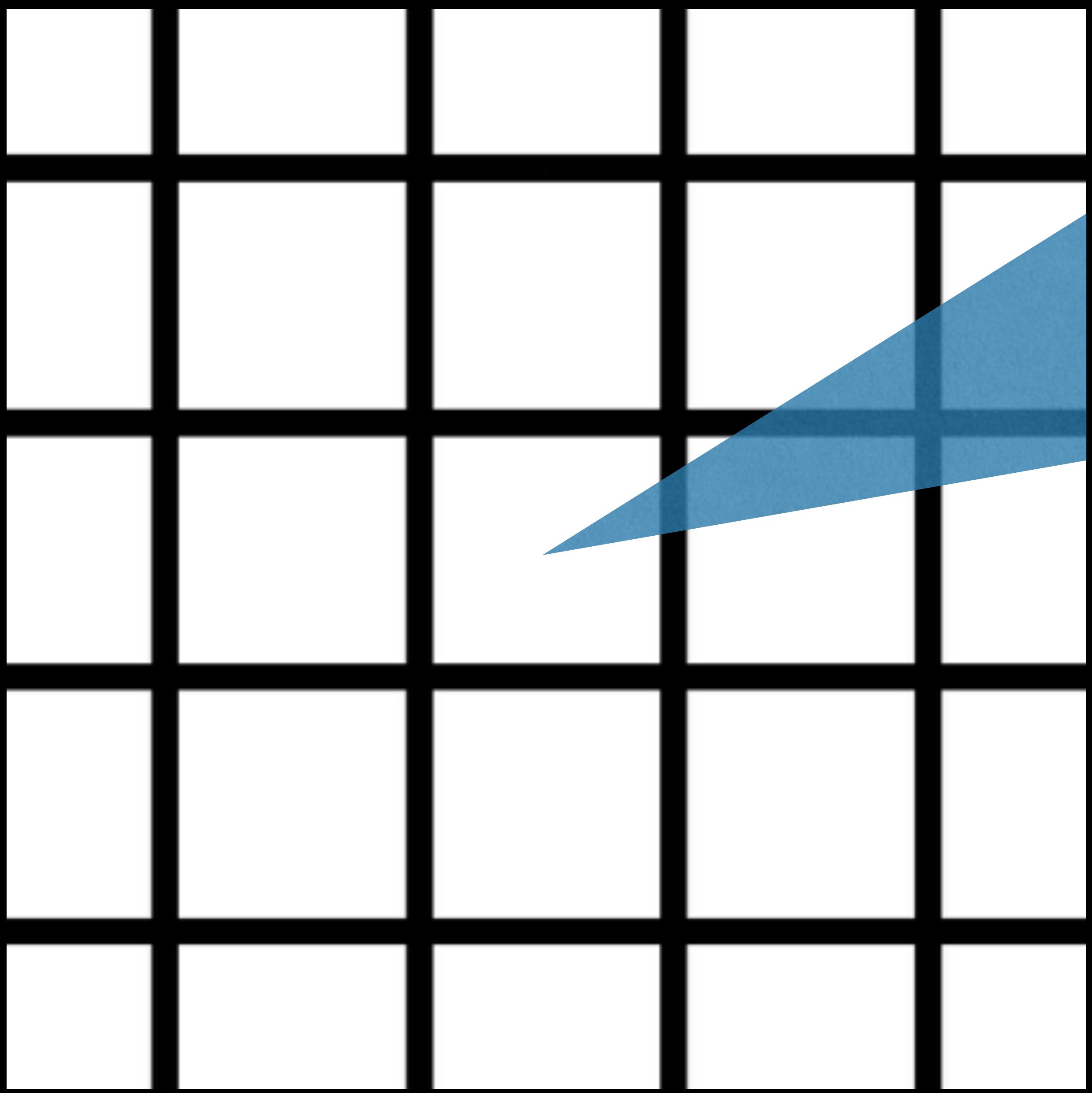
Orthographic projection onto the w=1 hypersurface $y_i = \dfrac{x_i}{w}$

The Poincaré Disk Model

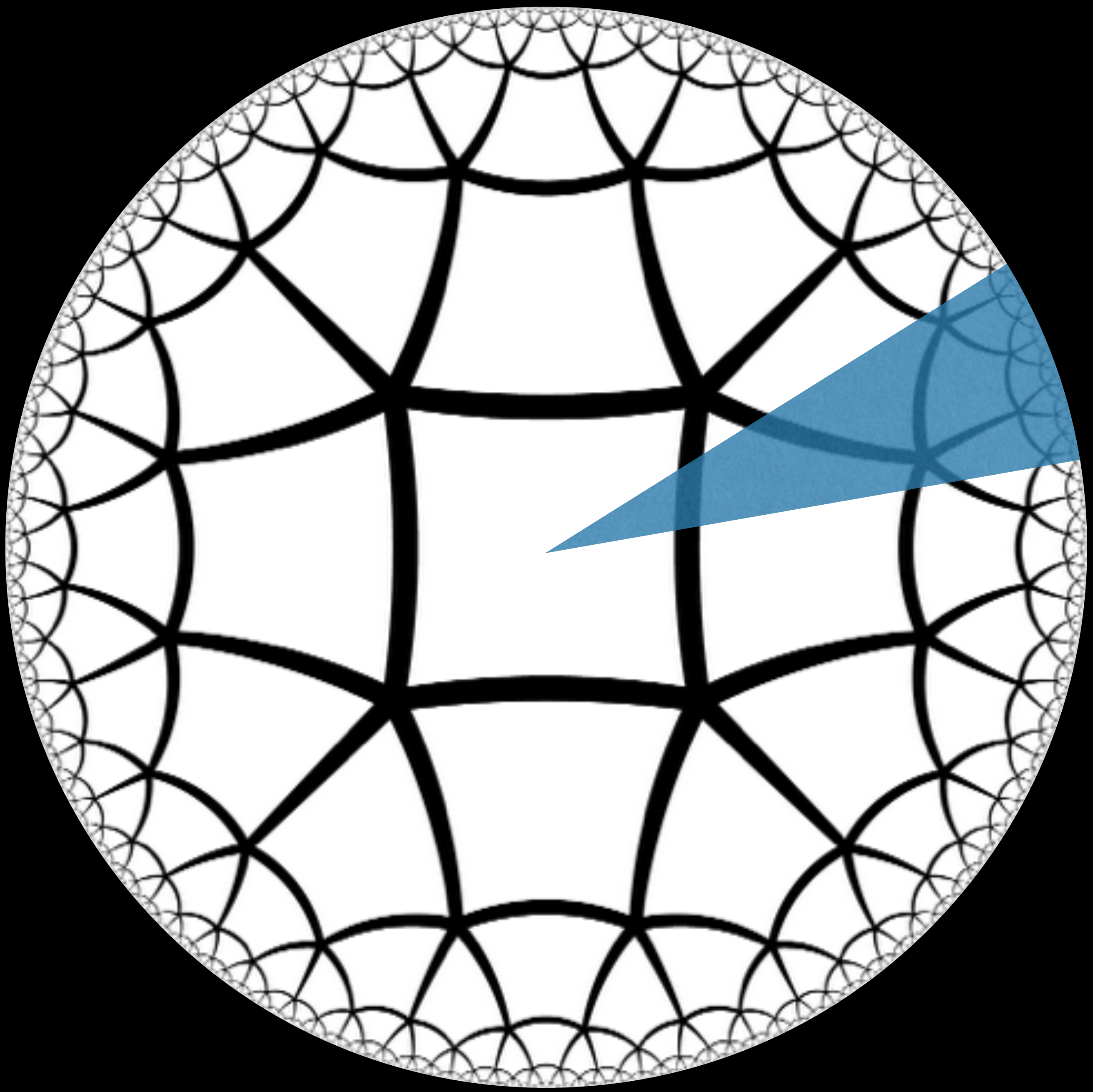
Stereographic projection from the point w=-1 onto the w=1 hypersurface

$$z_i = \frac{x_i}{1+w}, \quad x_1 = \frac{(1 + \sum z_i^2, 2z_i)}{1 - \sum z_i^2}$$

Seeing via geodesics



{4,4}



{4,5}

Render by Roice Nelson

Moving in $\mathbb{H}^3$

Isometries, the exponential map and GPUs

Isometries of $\mathbb{H}^3$ are elements of $\mathrm{SO}^+(3,1)$.

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & dx \\ 0 & 0 & 0 & dy \\ 0 & 0 & 0 & dz \\ dx & dy & dz & 0 \end{pmatrix}$$

$$\mathbf{M}^3 = |d\mathbf{r}|^2\mathbf{M}, \quad \mathbf{M}^4 = |d\mathbf{r}|^2\mathbf{M}^2$$

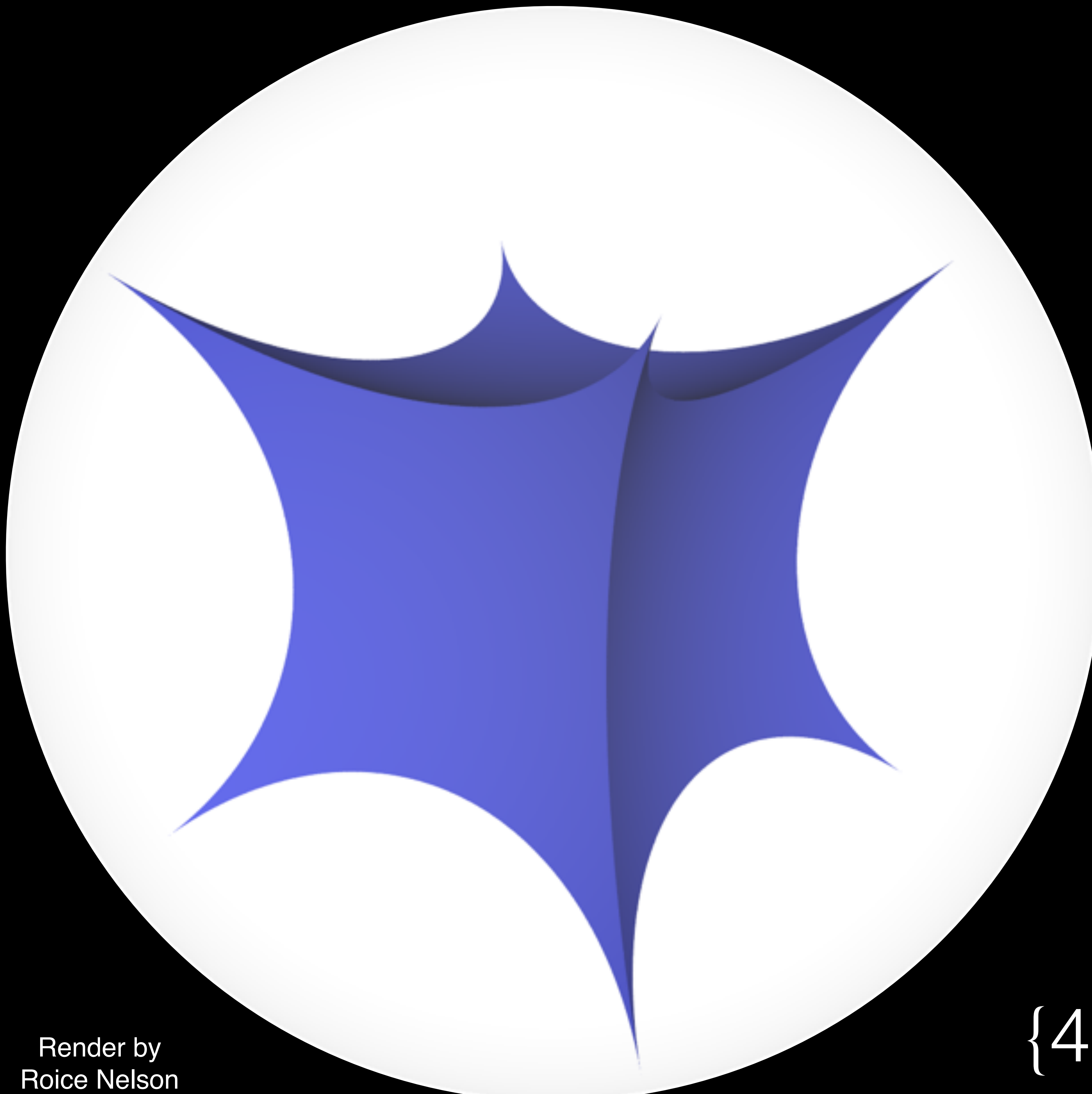
$$|d\mathbf{r}| = \sqrt{dx^2 + dy^2 + dz^2}$$

$$\exp\mathbf{M} = \mathbf{Id} + \frac{\sinh(|d\mathbf{r}|)}{|d\mathbf{r}|}\mathbf{M} + \frac{\cosh(|d\mathbf{r}|) - 1}{|d\mathbf{r}|^2}\mathbf{M}^2$$

Update points on the screen according to:

$$\mathbf{r}(t+\Delta t) = \exp\mathbf{M}(\mathbf{r}(\Delta t))\,\mathbf{r}(t)$$

What to draw in $\mathbb{H}^3$



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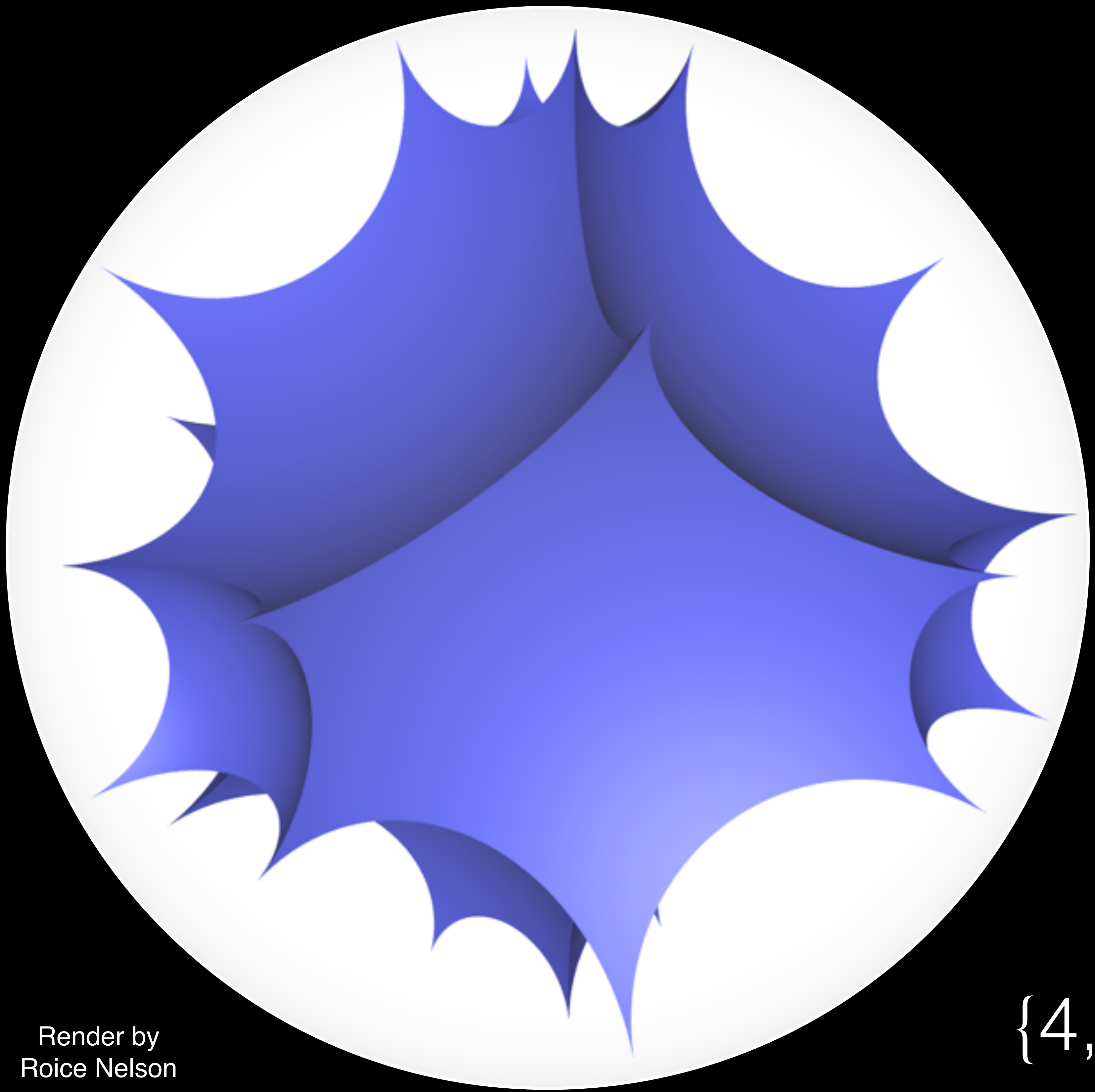
{4,3,6}

What to draw in $\mathbb{H}^3$



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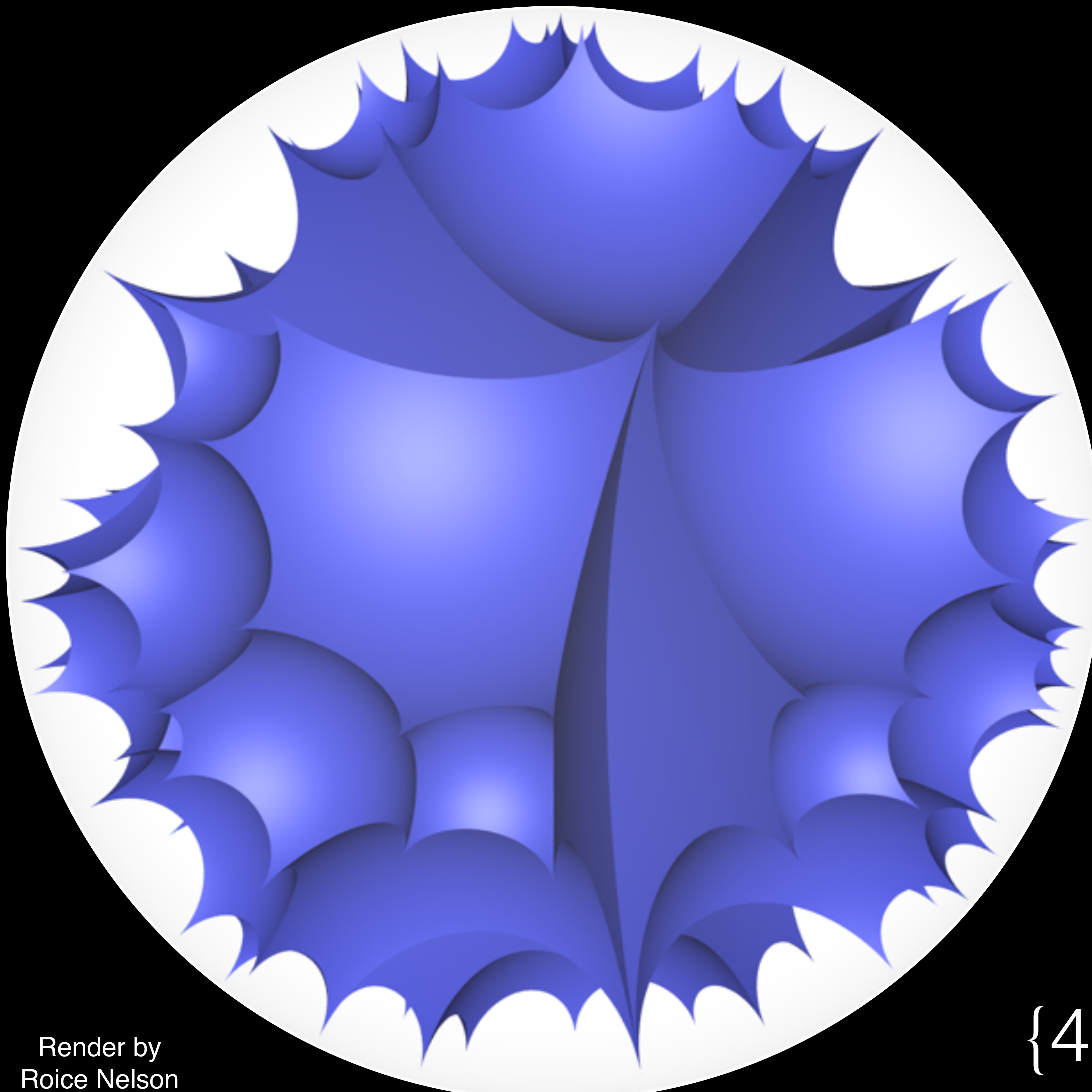
What to draw in $\mathbb{H}^3$



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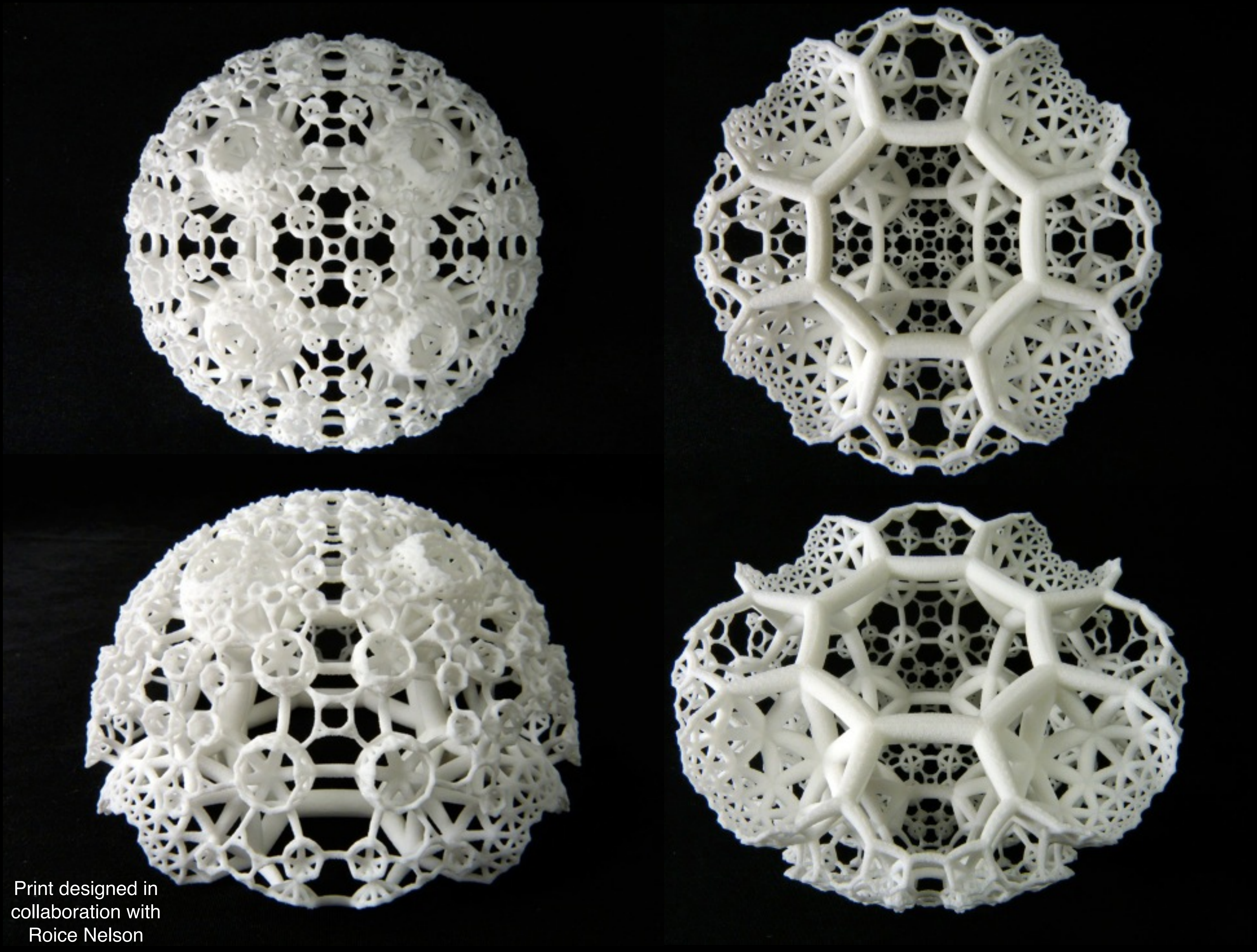
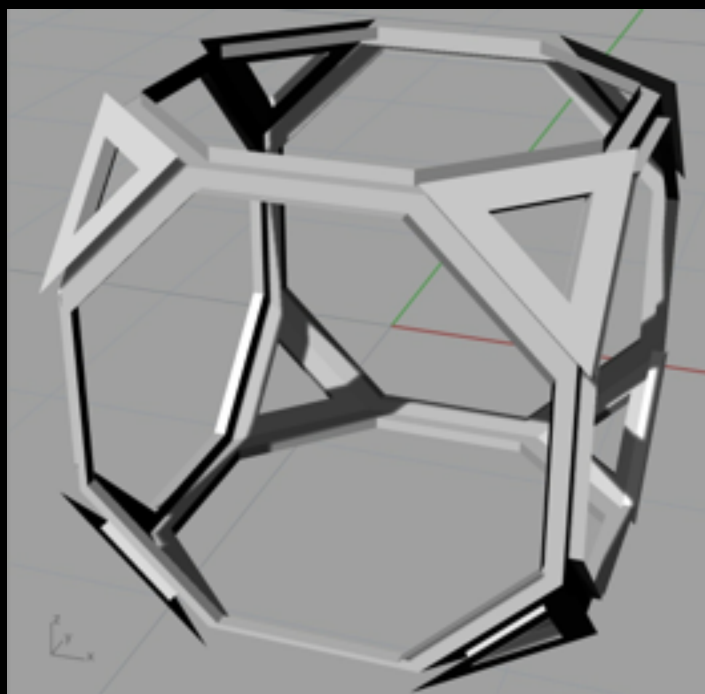


Print designed in collaboration with Roice Nelson

Drawing $\mathbb{H}^3$ on screen

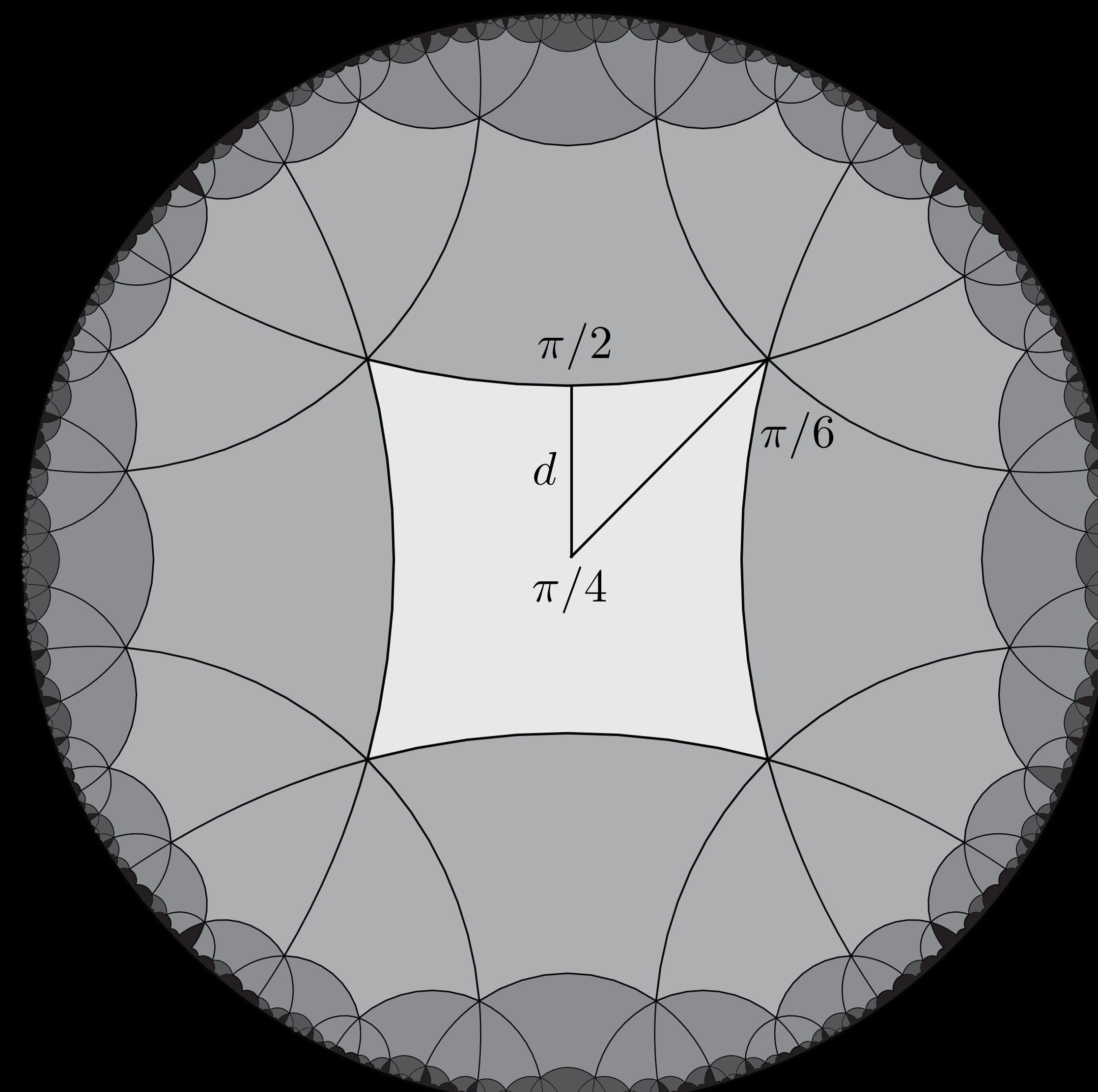


We aren't seeing points directly in $\mathbb{H}^3$, instead we view their image in the tangent space at the point {0,0,0,1} on the hyperboloid.

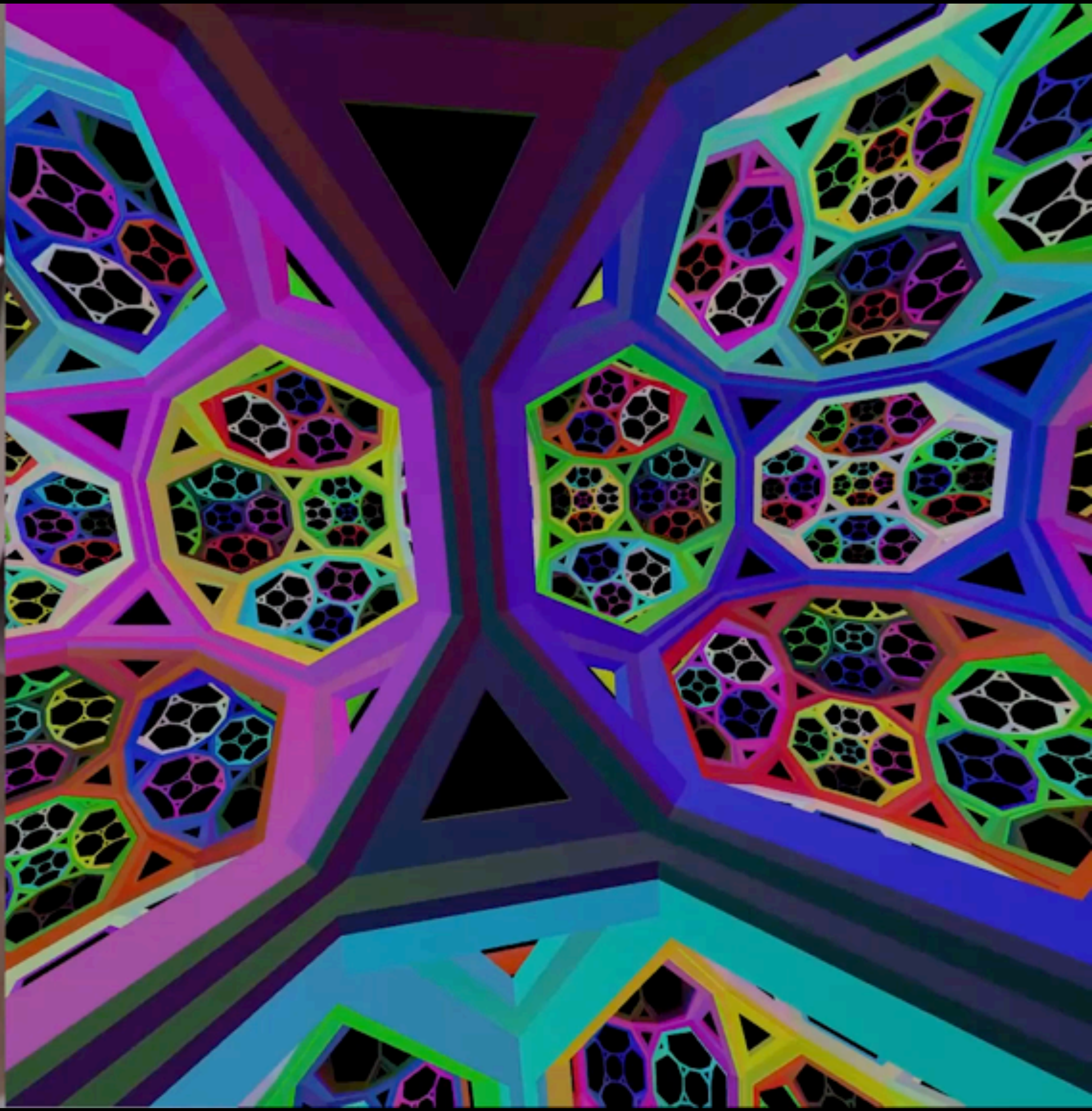
Project points from $T_p\mathbb{H}^3 \in \mathbb{E}^3$ to the hyperboloid using the Klein model.

$$\cosh(d) = \frac{\cos(\pi/6)}{\sin(\pi/4)}$$

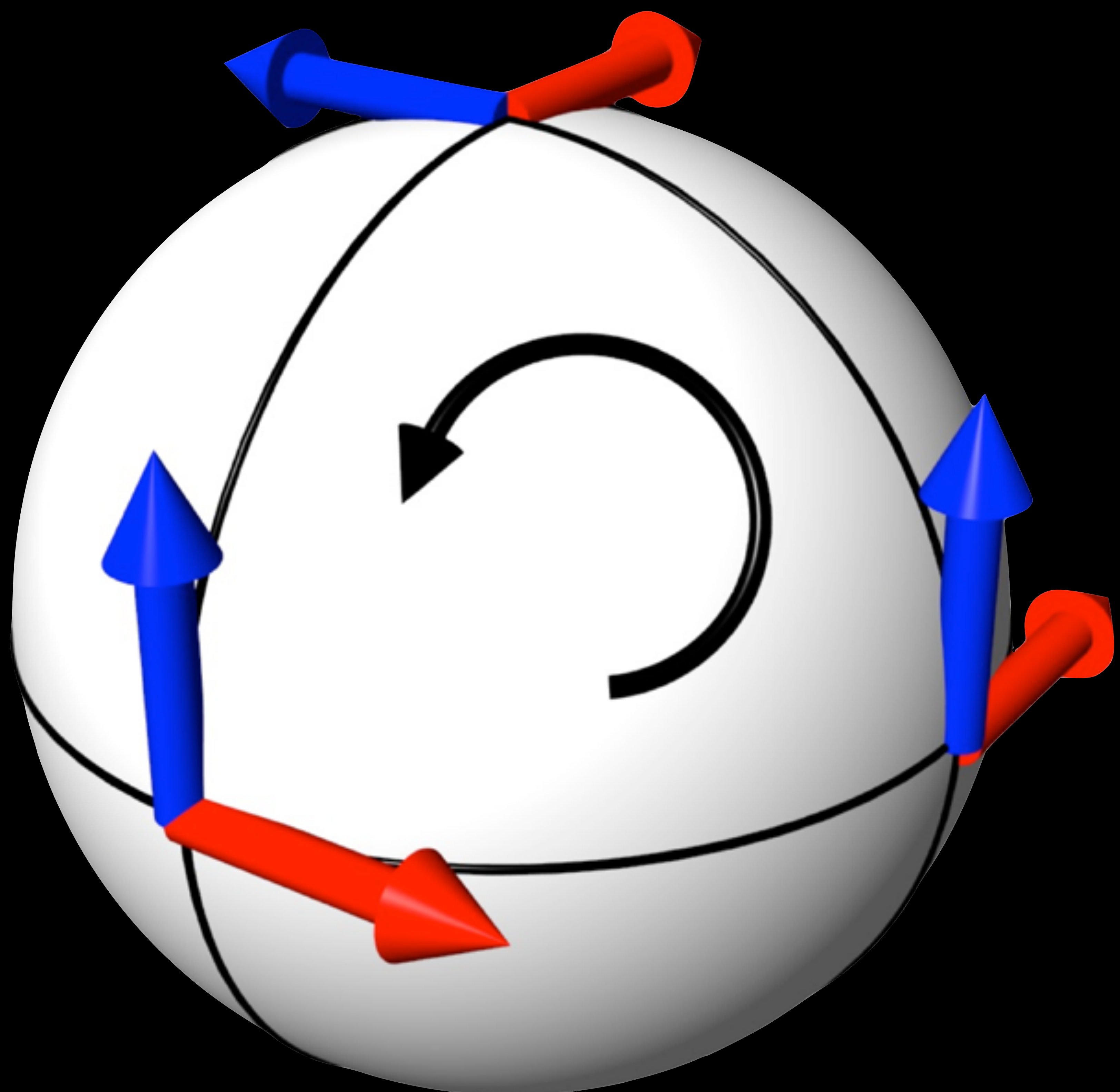
Each vertex in the .obj file is acted upon by exp(**M**) every time the user moves.



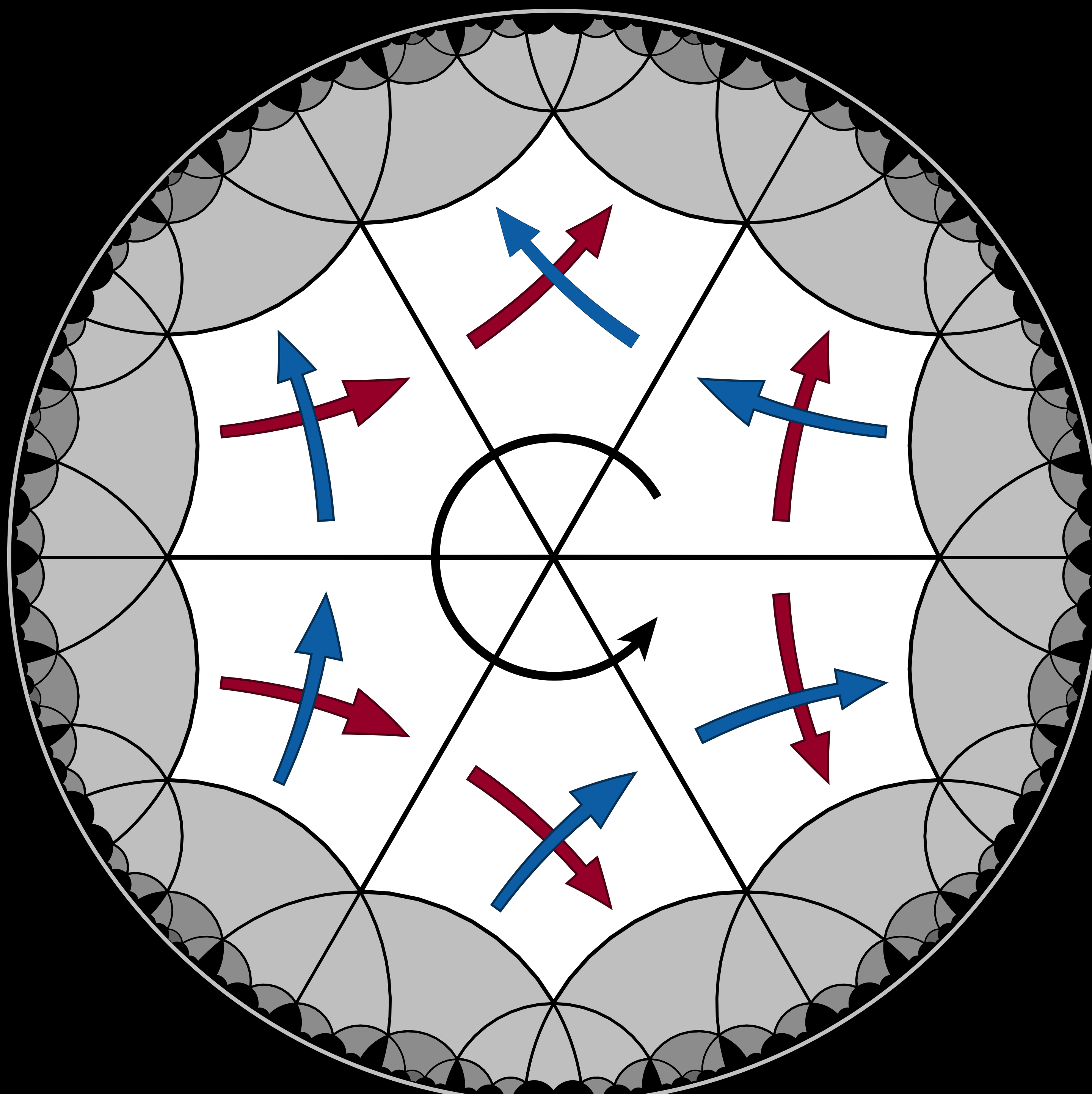
Hyperbolic space: six cubes



Parallel transport and Holonomy



Parallel transport and Holonomy

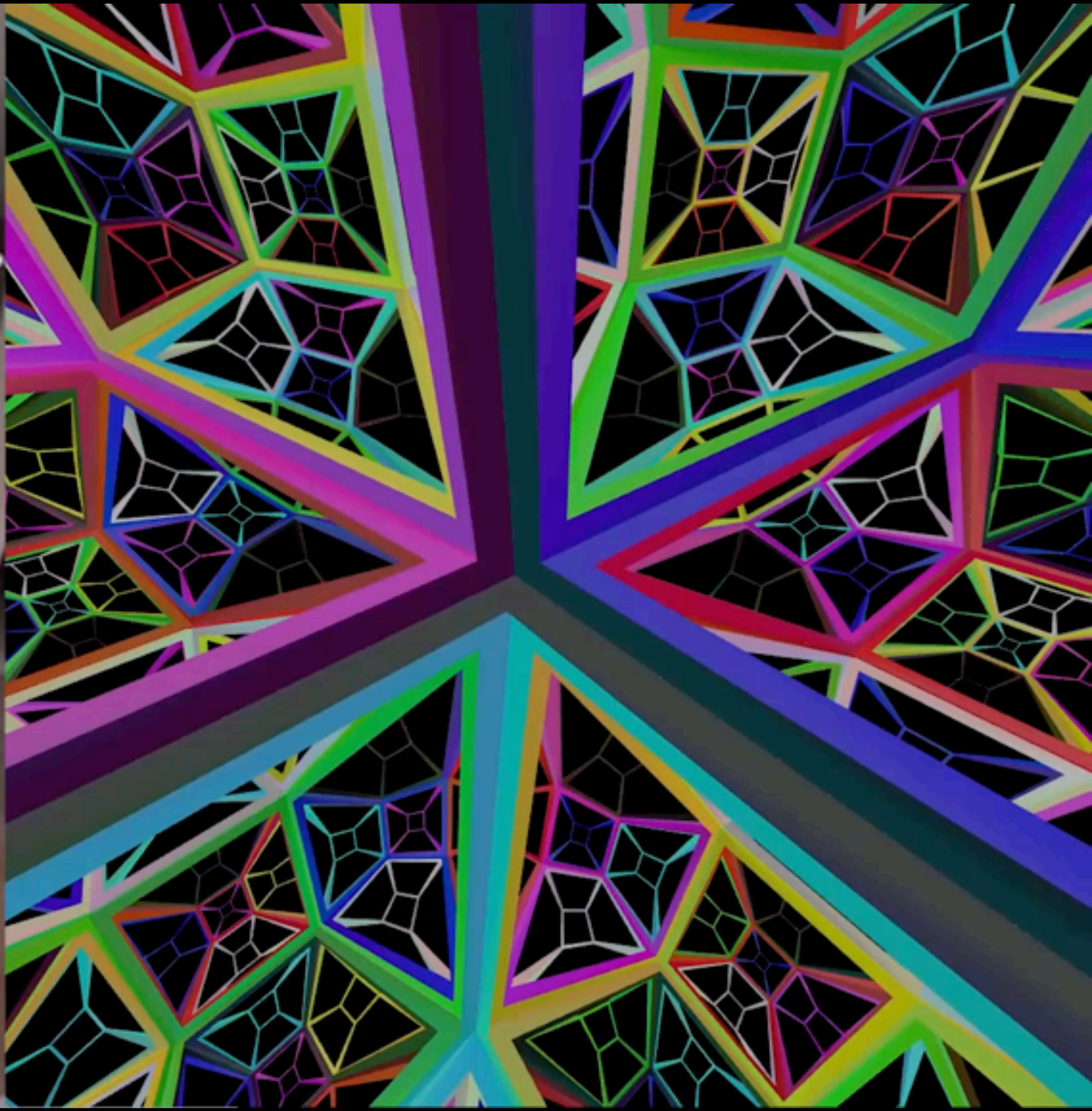


Demos

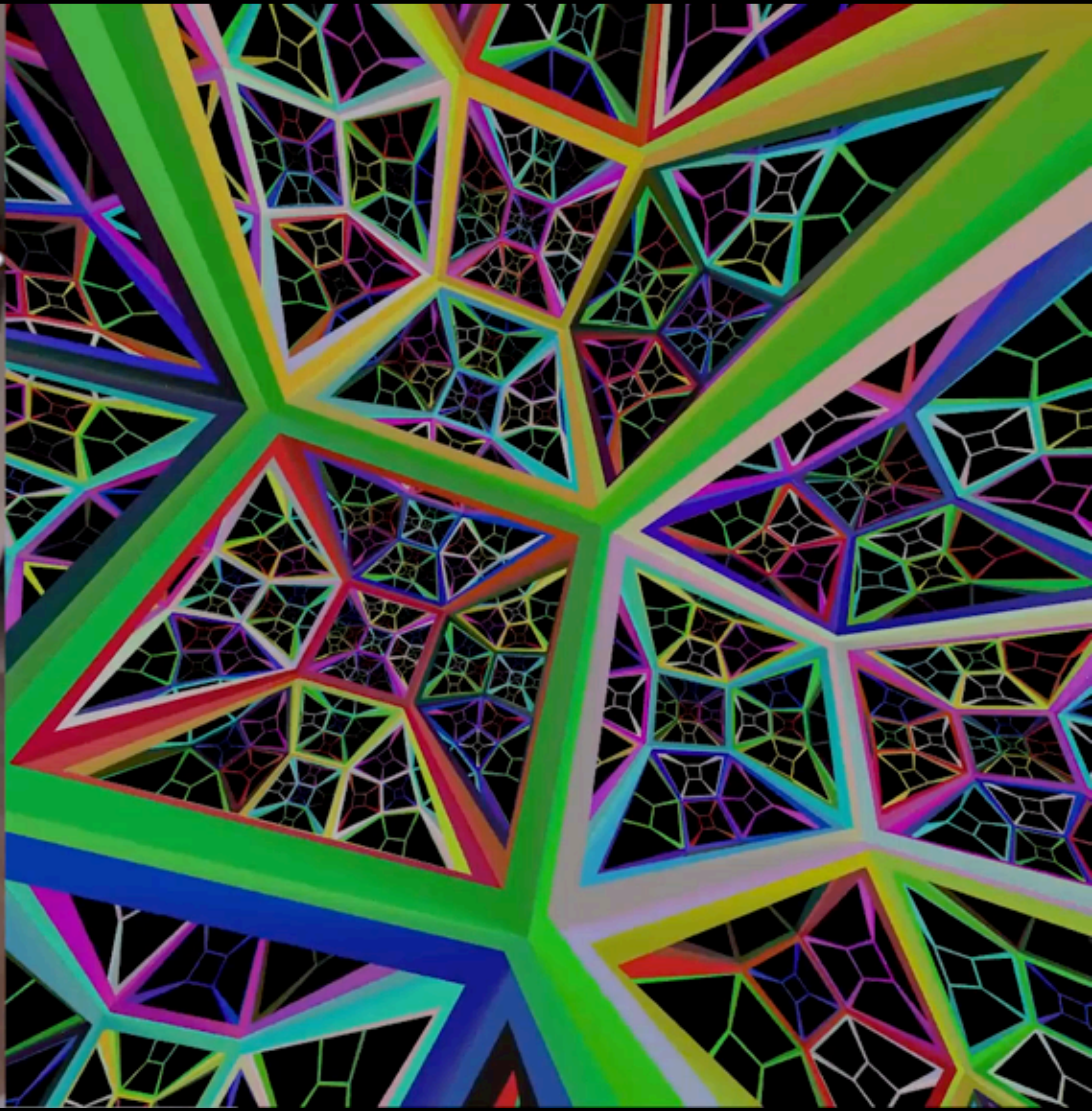
1. Is that an icosahedron?
2. Cube vertices
3. Holonomy
4. Yet more holonomy
5. Monkeys!

Inside the not-icosahedra

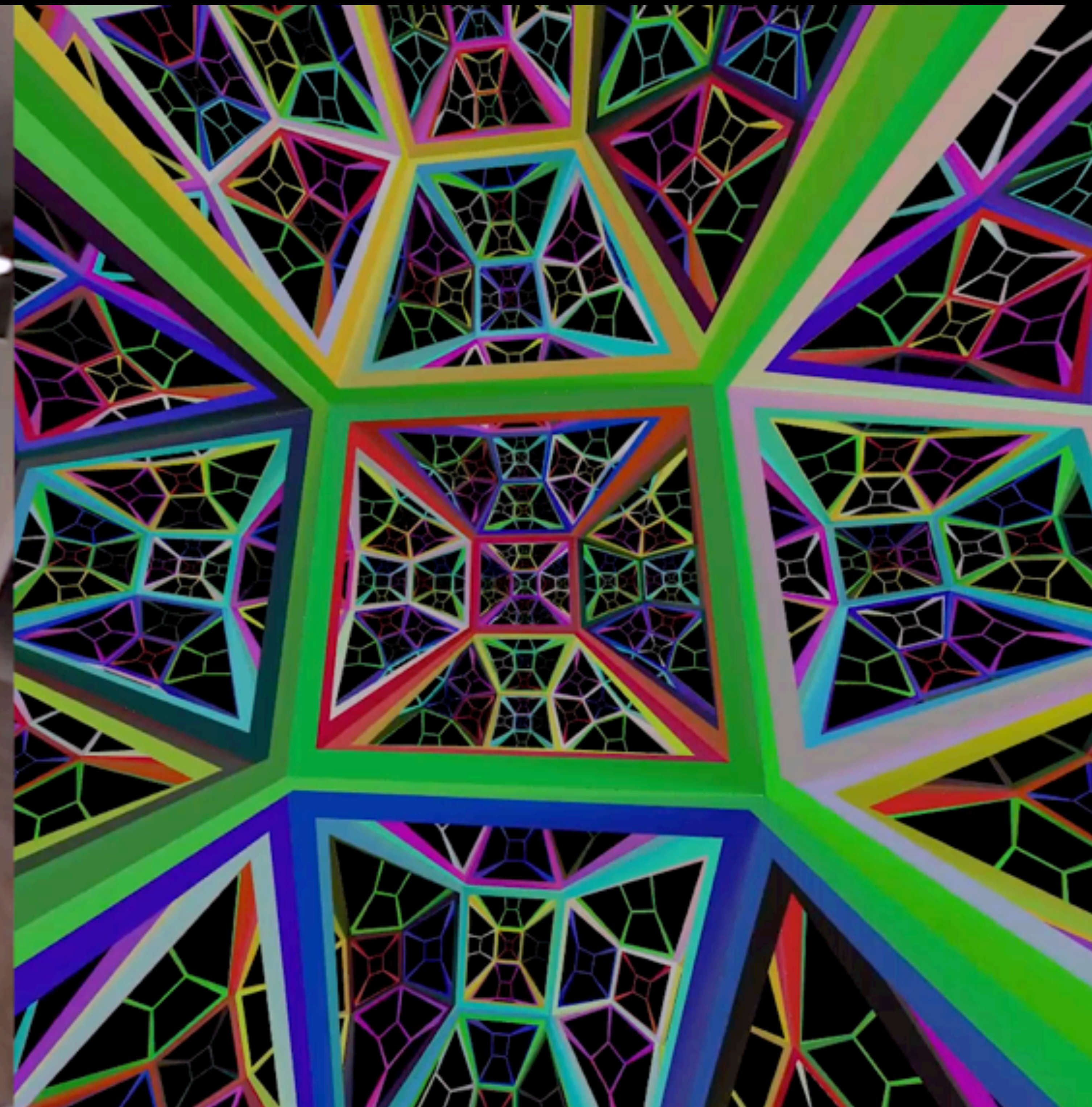
Cube vertices



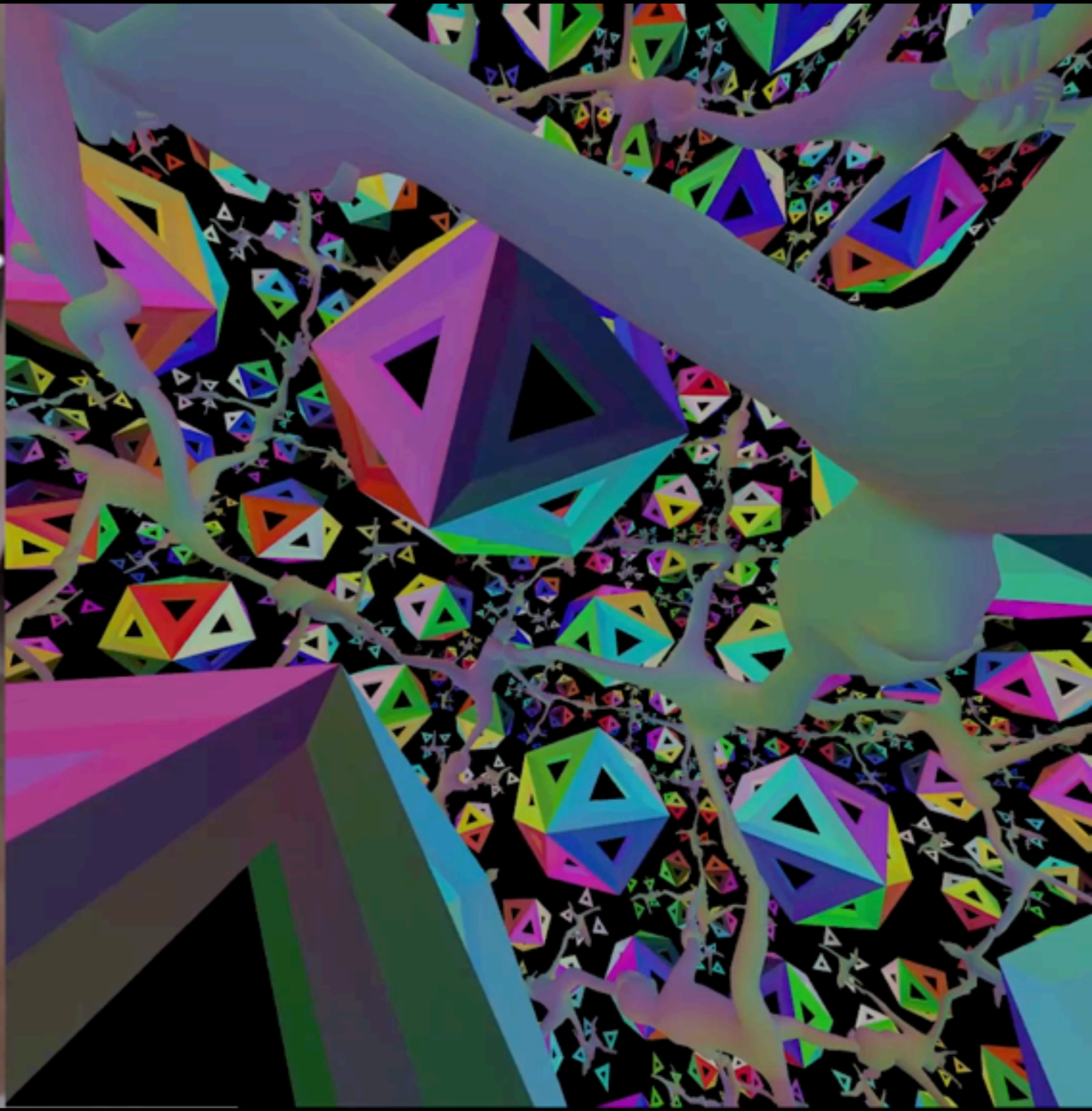
Parallel Transport and Holonomy

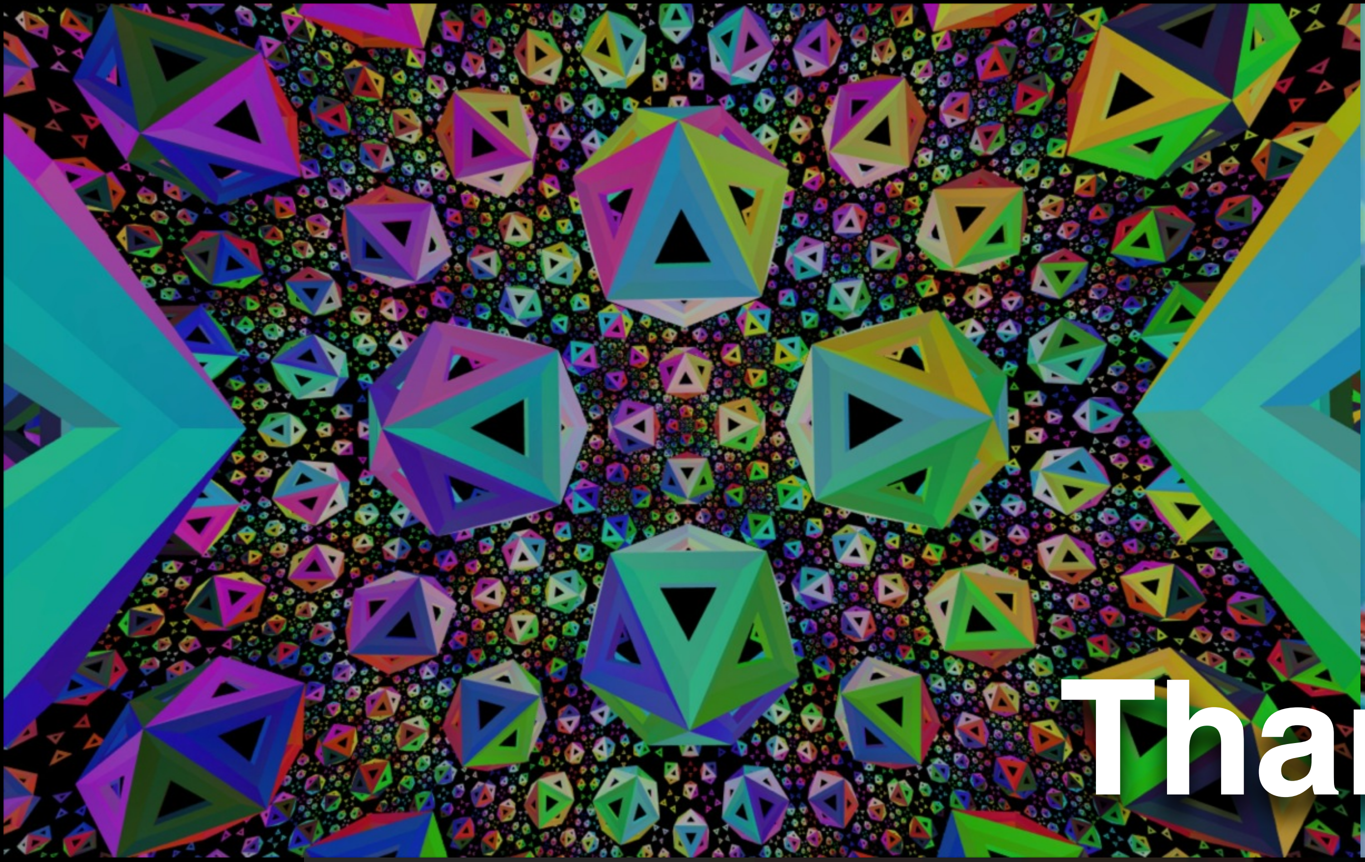



Yet more Holonomy



Monkeys





Thanks!