Kirigami: Using Cutting, Glueing and Folding to Refurbish the Plane

Toen Castle, UPenn

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partially published in:
Science Advances, 2016
some other high profile journal in a month or so

SIMONS FOUNDATION

Kirigami is like **origami**, but with cutting and (for us) rejoining of the paper.

Note the fiddly little folds, much smaller than the structural features.

Origami

ORIGIN Japanese, from ***oru***, ***-ori 'fold'*** + ***kami 'paper'***.

paper rabbit by Eric Demaine and Tomohiro Tachi.

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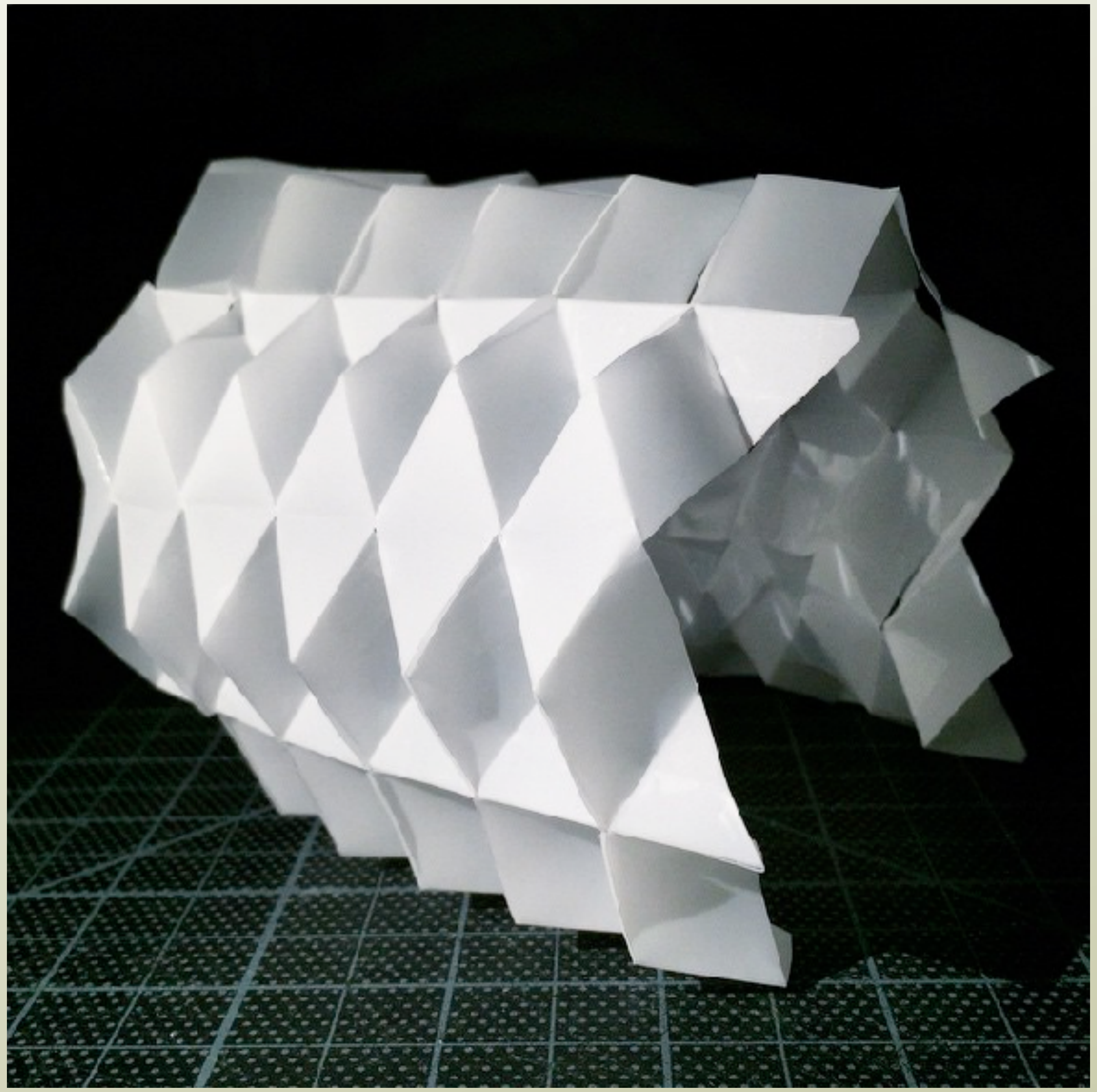
Kirigami



ORIGIN Japanese, from ***-kiri* ‘cut’** + ***kami* ‘paper’**.

Image by Michael Tanis
instagram.com/hyperqbert
www.flickr.com/photos/miketanis

The cuts and folds are on the same scale as the structural features.

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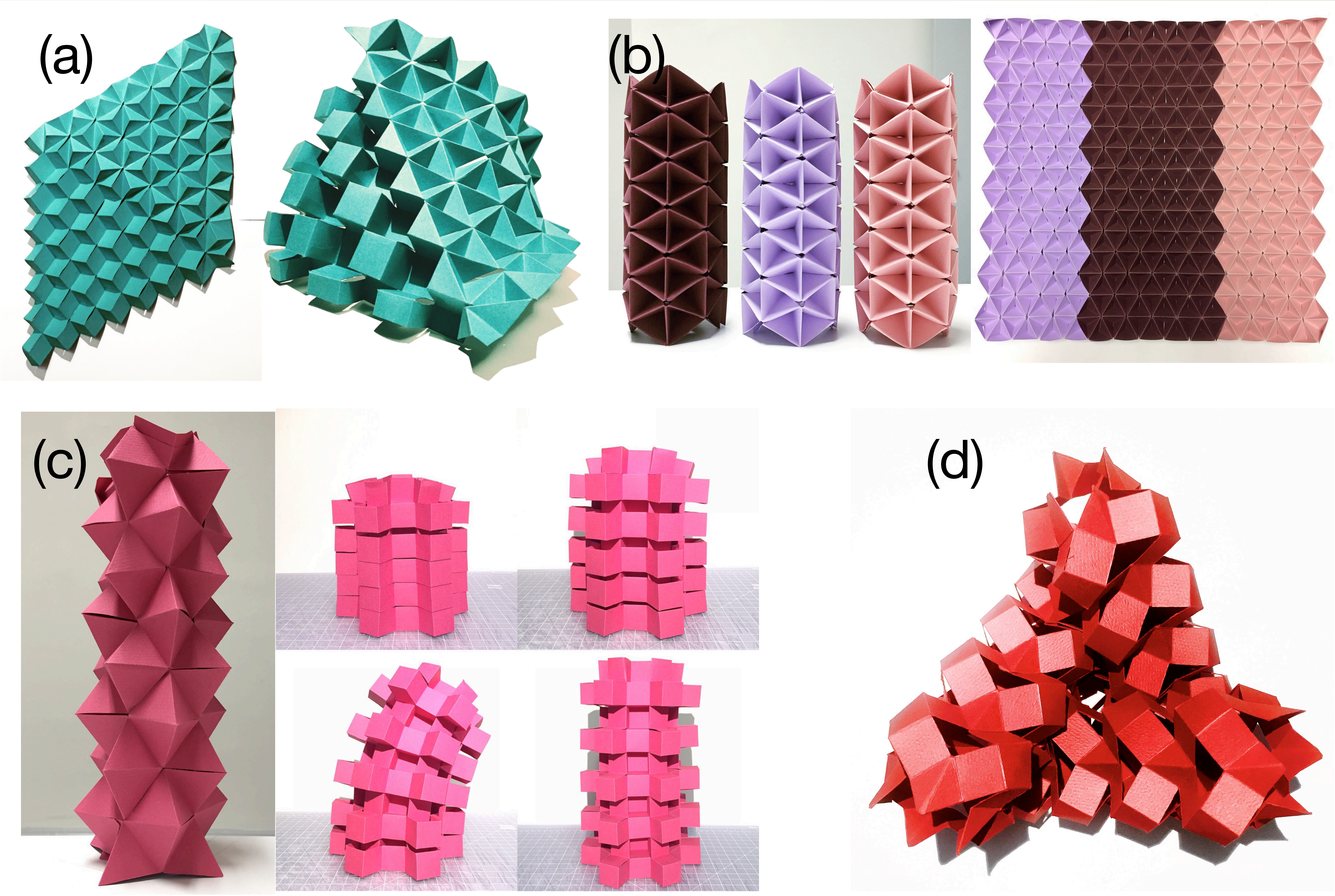
Kirigami



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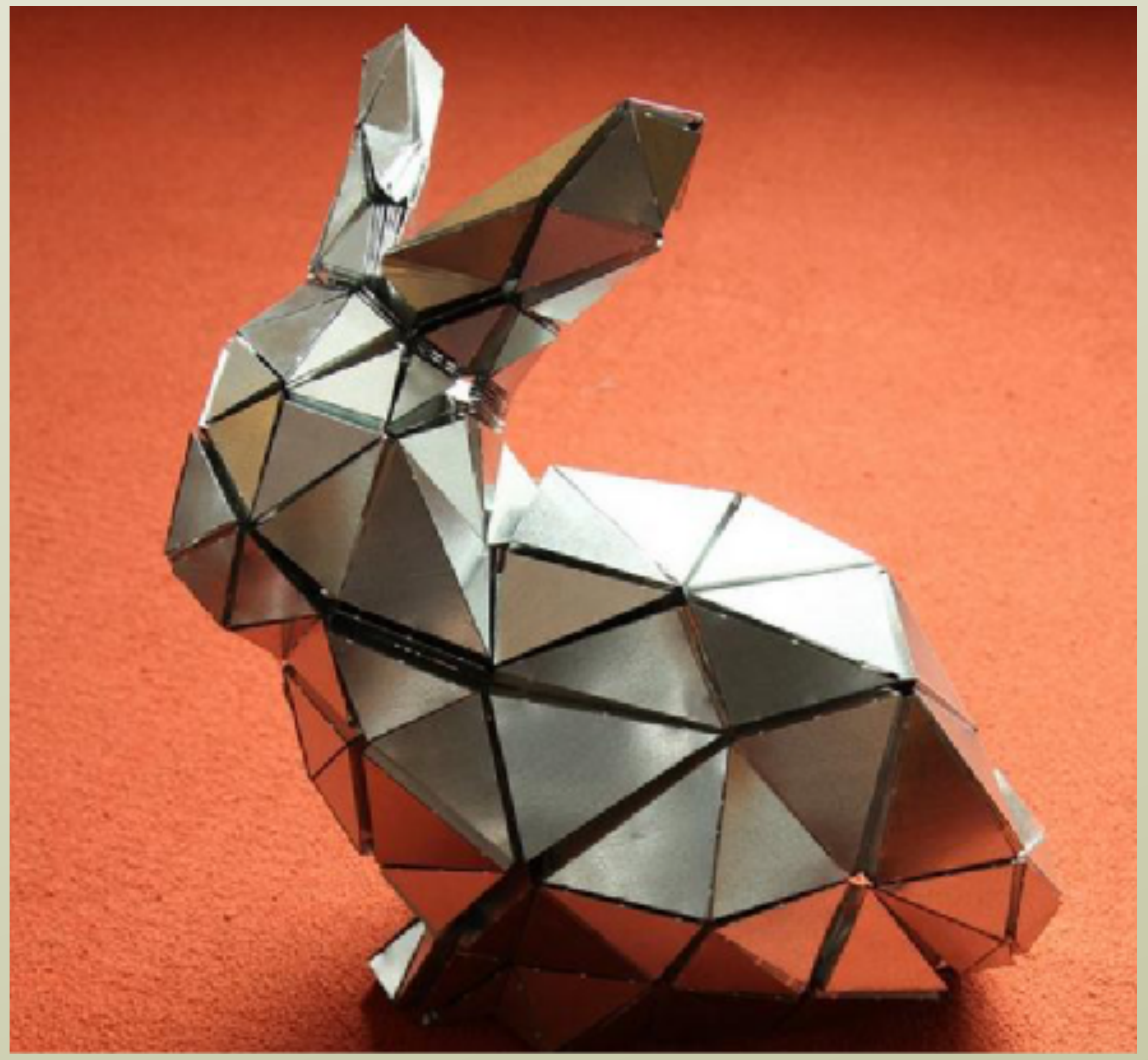
Kirigami produces structures more simply than **origami**, as there is no need for elaborate folds to mimic Gaussian curvature.



From an *origami* perspective, the most obvious use for cutting is to remove extraneous material that would have had to be tucked out of sight.

But *kirigami* offers more than this.

metal rabbit by Eric Demaine and Tomohiro Tachi.

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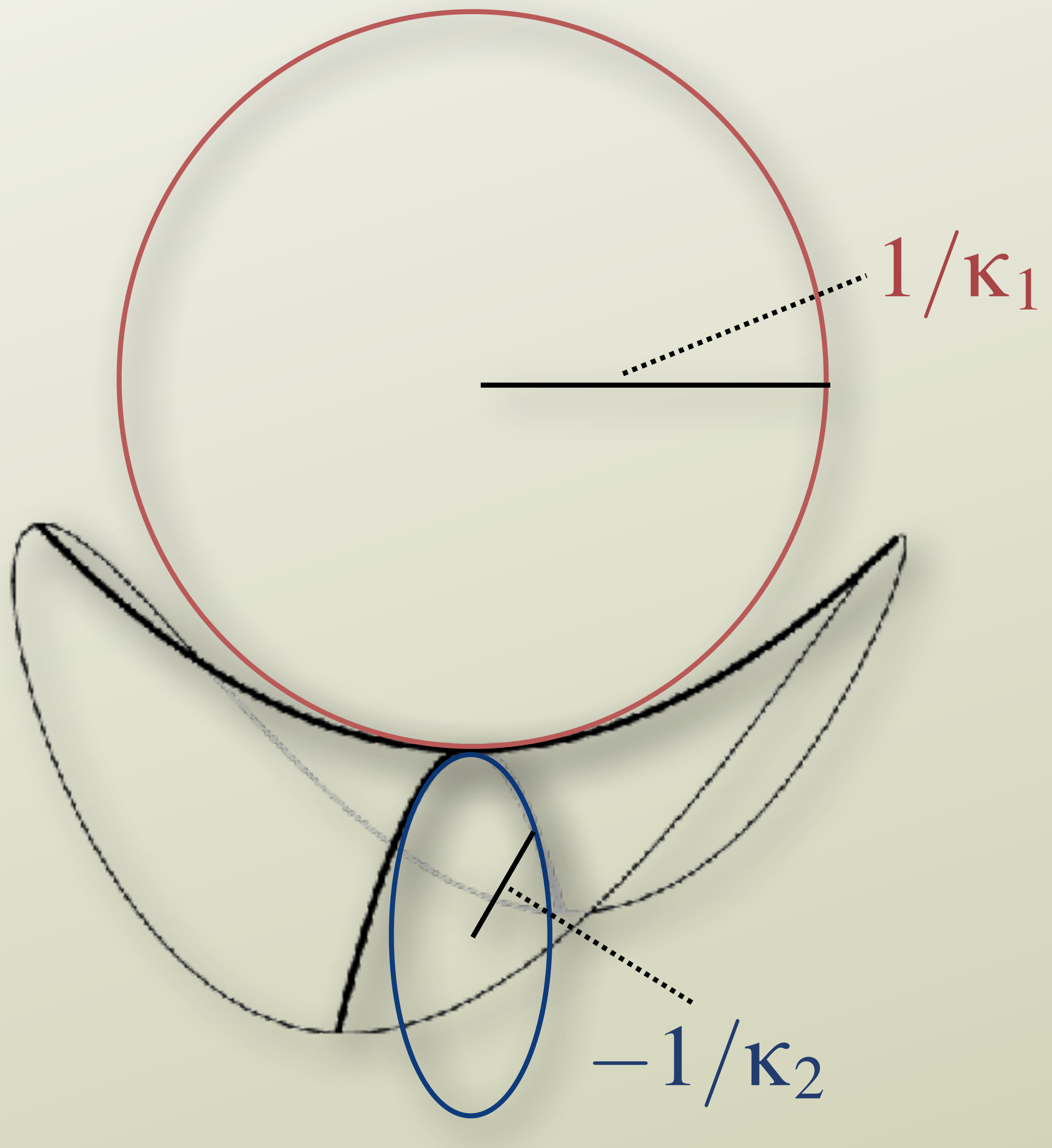
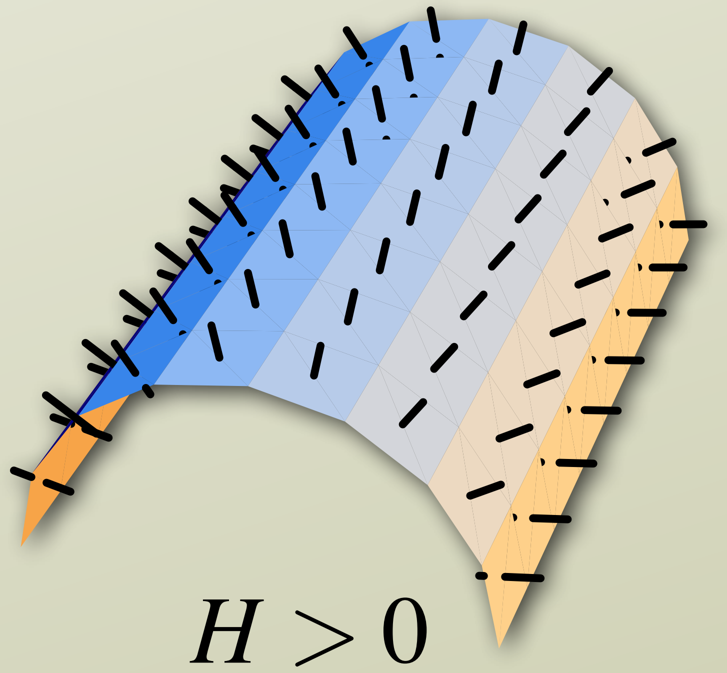
Two Kinds of Curvature



Mean or Extrinsic

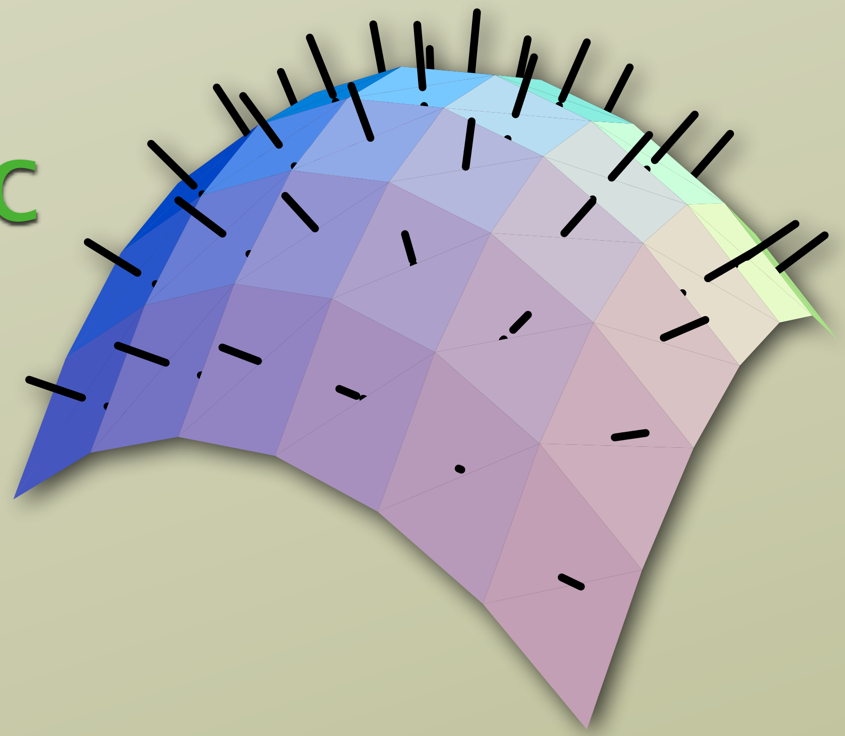
$$H = \frac{1}{2}(\kappa_1 + \kappa_2)$$



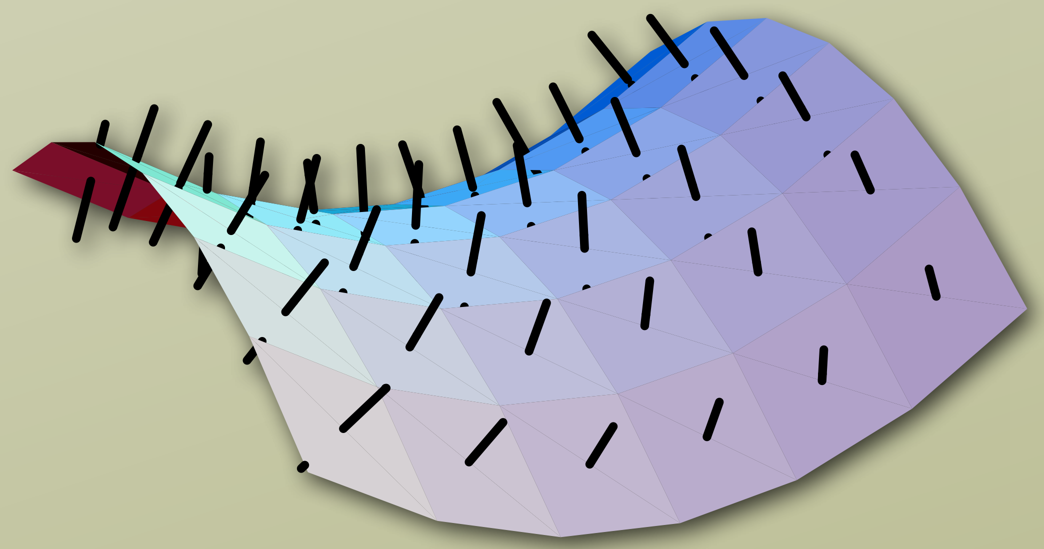
$H > 0$
$K = 0$

Gaussian or Intrinsic

$$K = \kappa_1 \kappa_2$$



$H > 0$
$K > 0$



$H = 0$
$K < 0$

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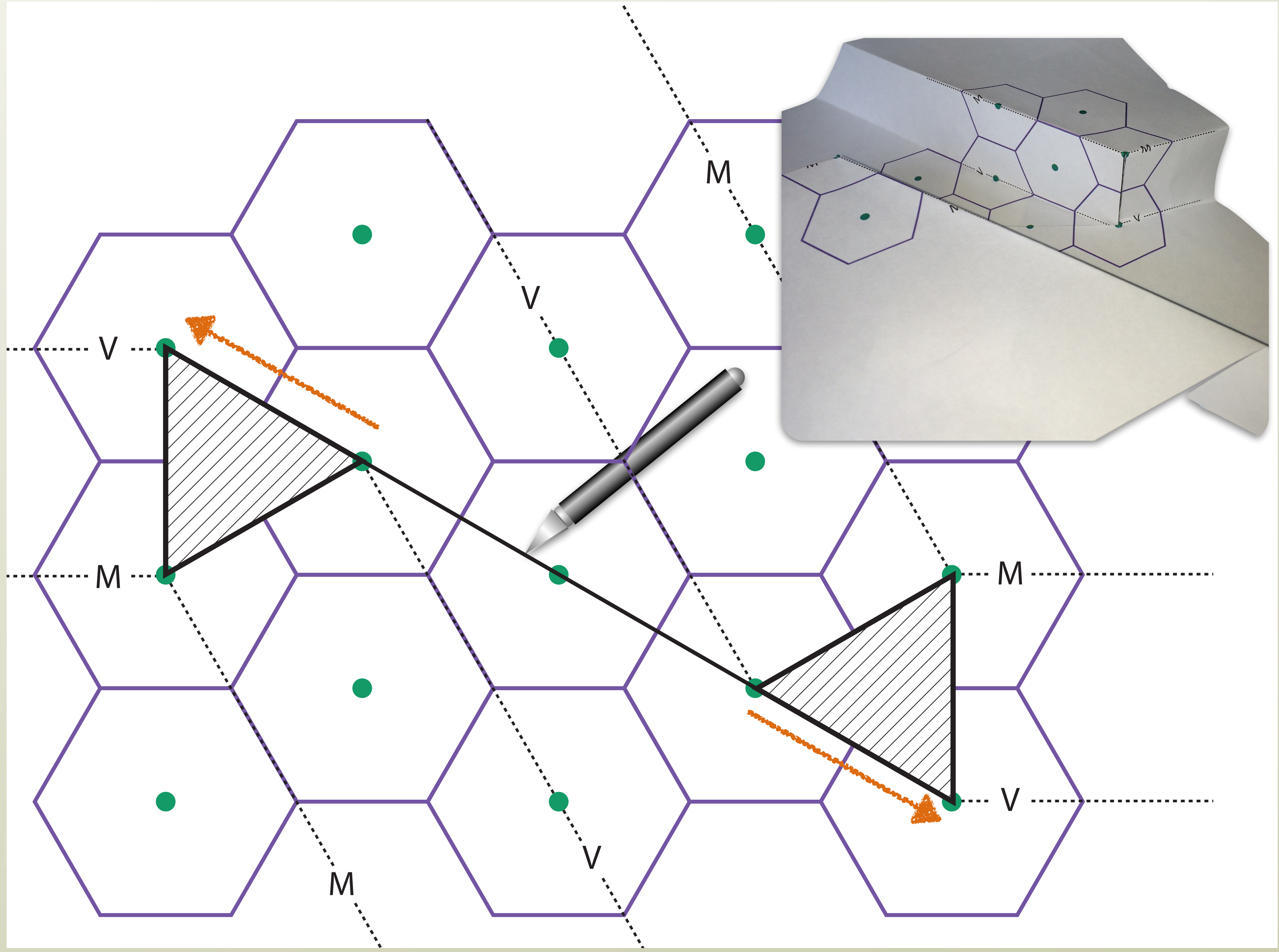
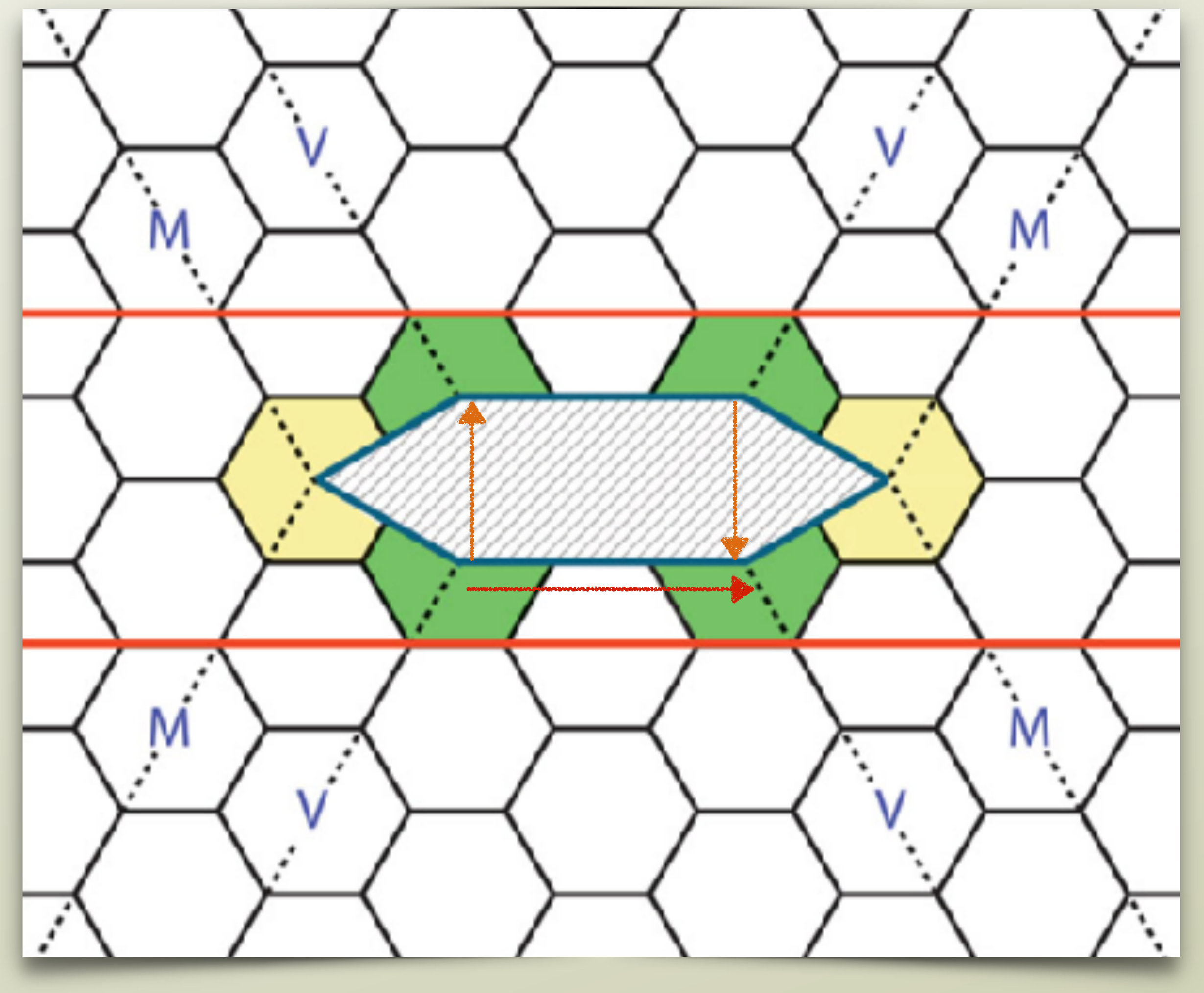
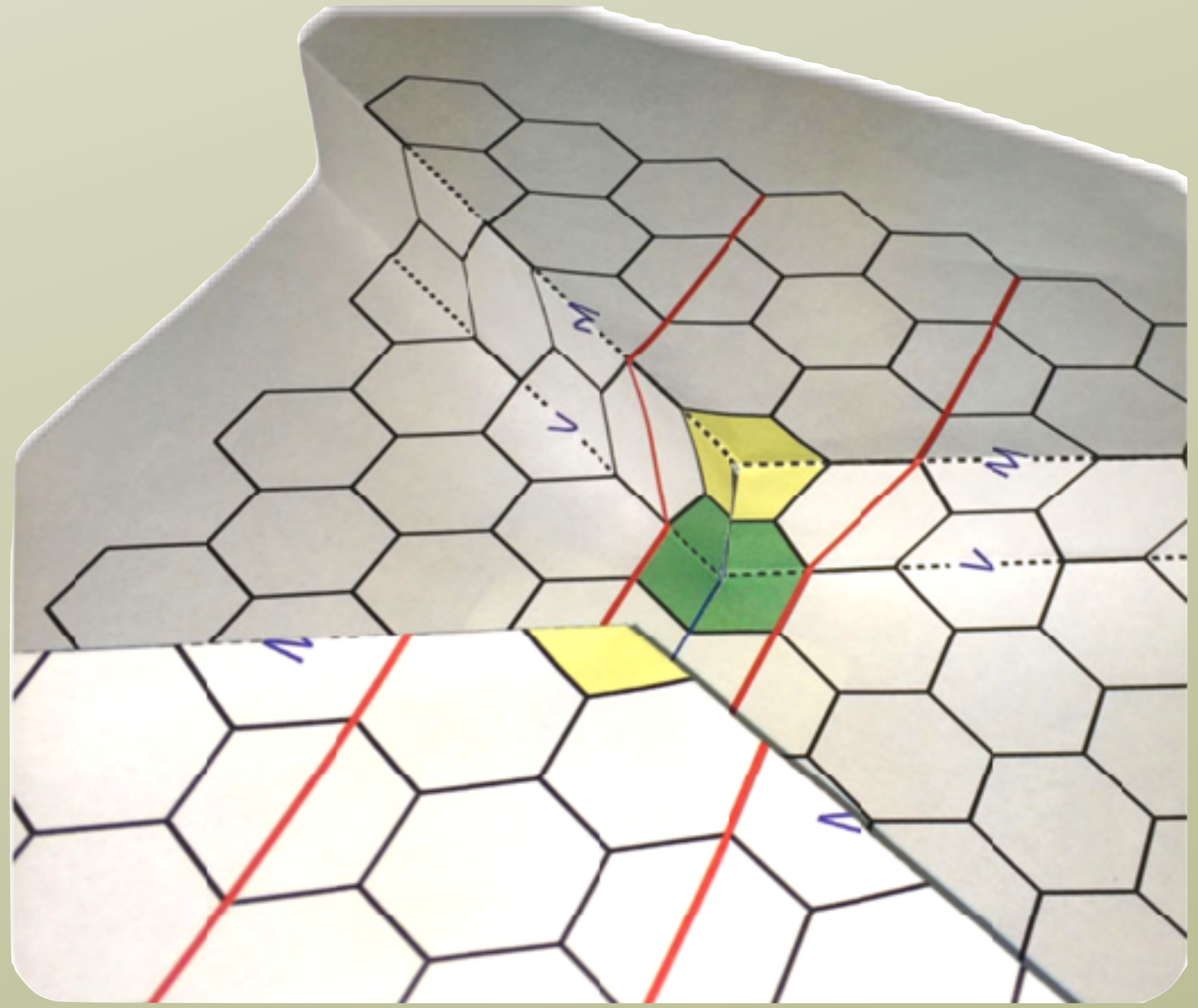
Old-school **Lattice Kirigami**:



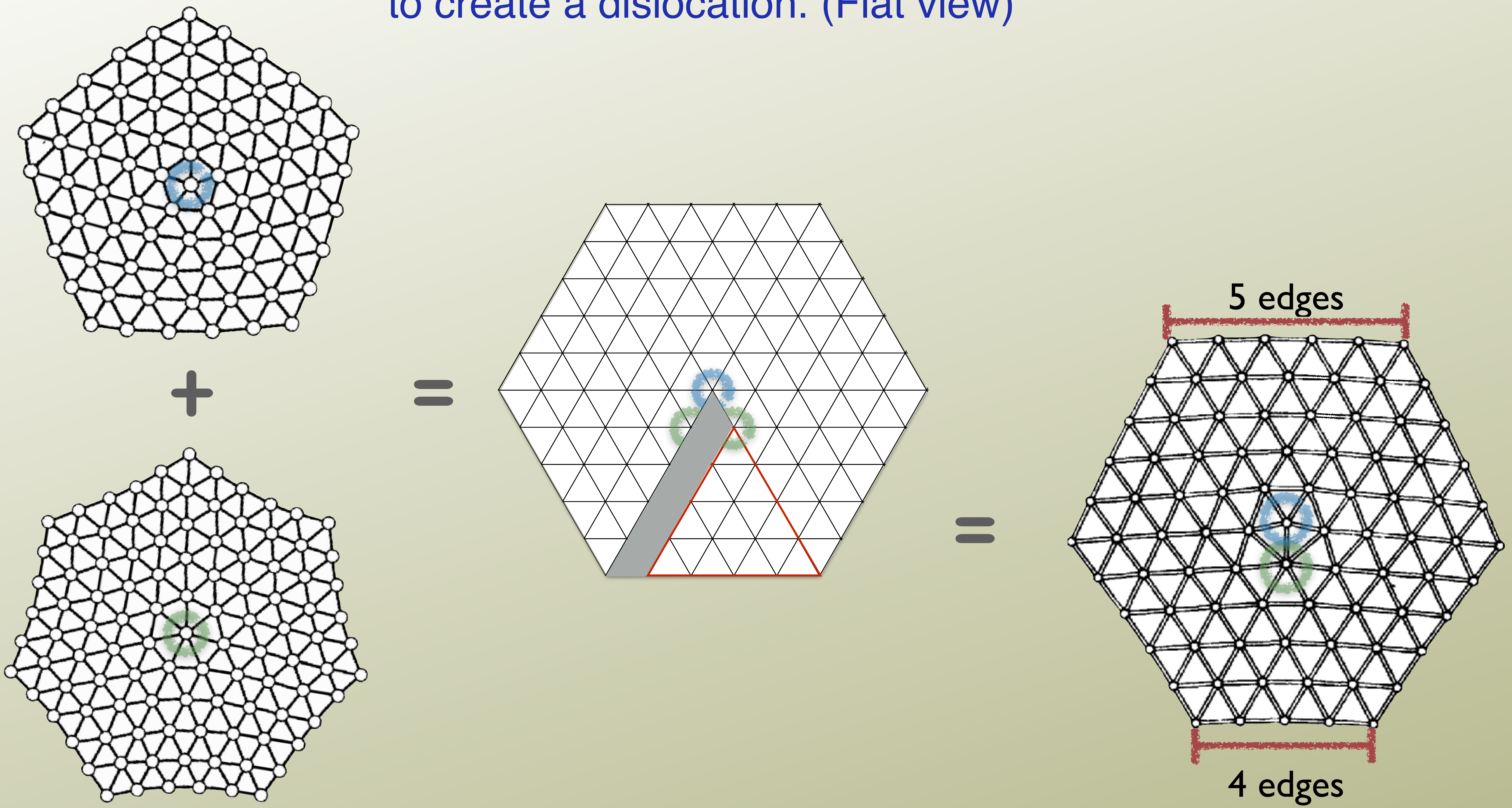




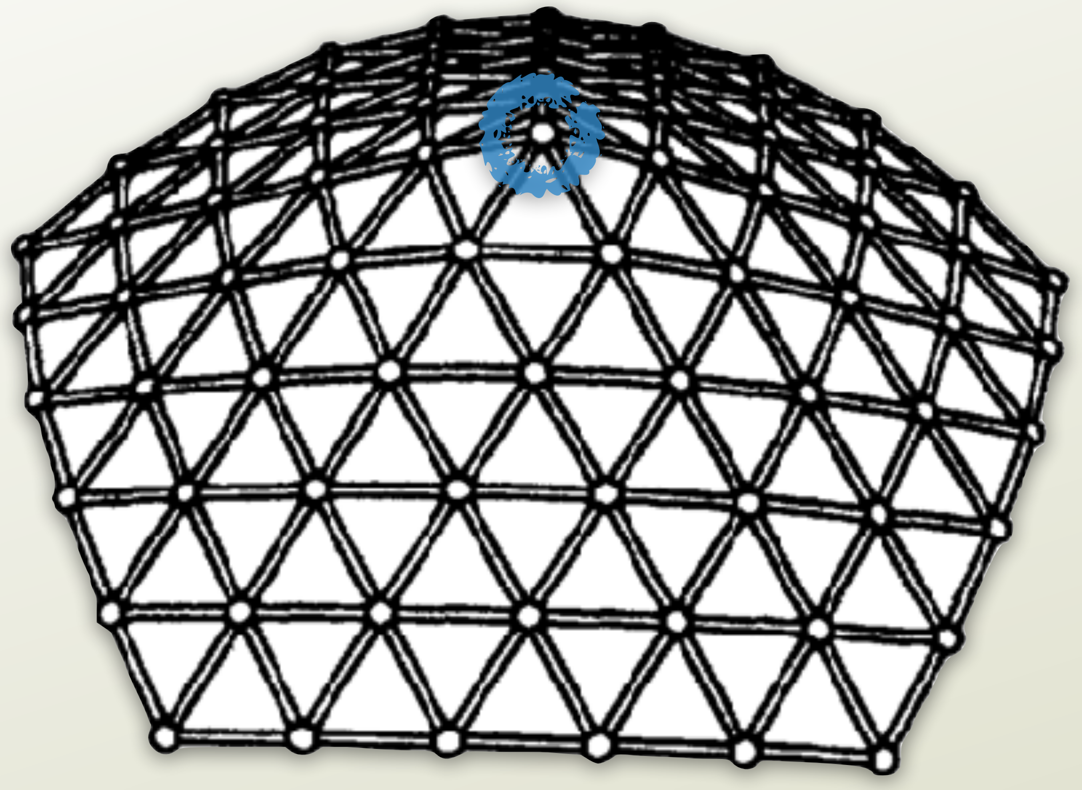
Burgers vectors climb

Making the Cut: Lattice Kirigami Rules
Phys. Rev. Lett. 113, 245502, 11 December 2014

Combining disclinations (points of non-zero Gaussian curvature) to create a dislocation. (Flat view)



Combining disclinations (points of non-zero Gaussian curvature) to create a dislocation.
(Buckled view)



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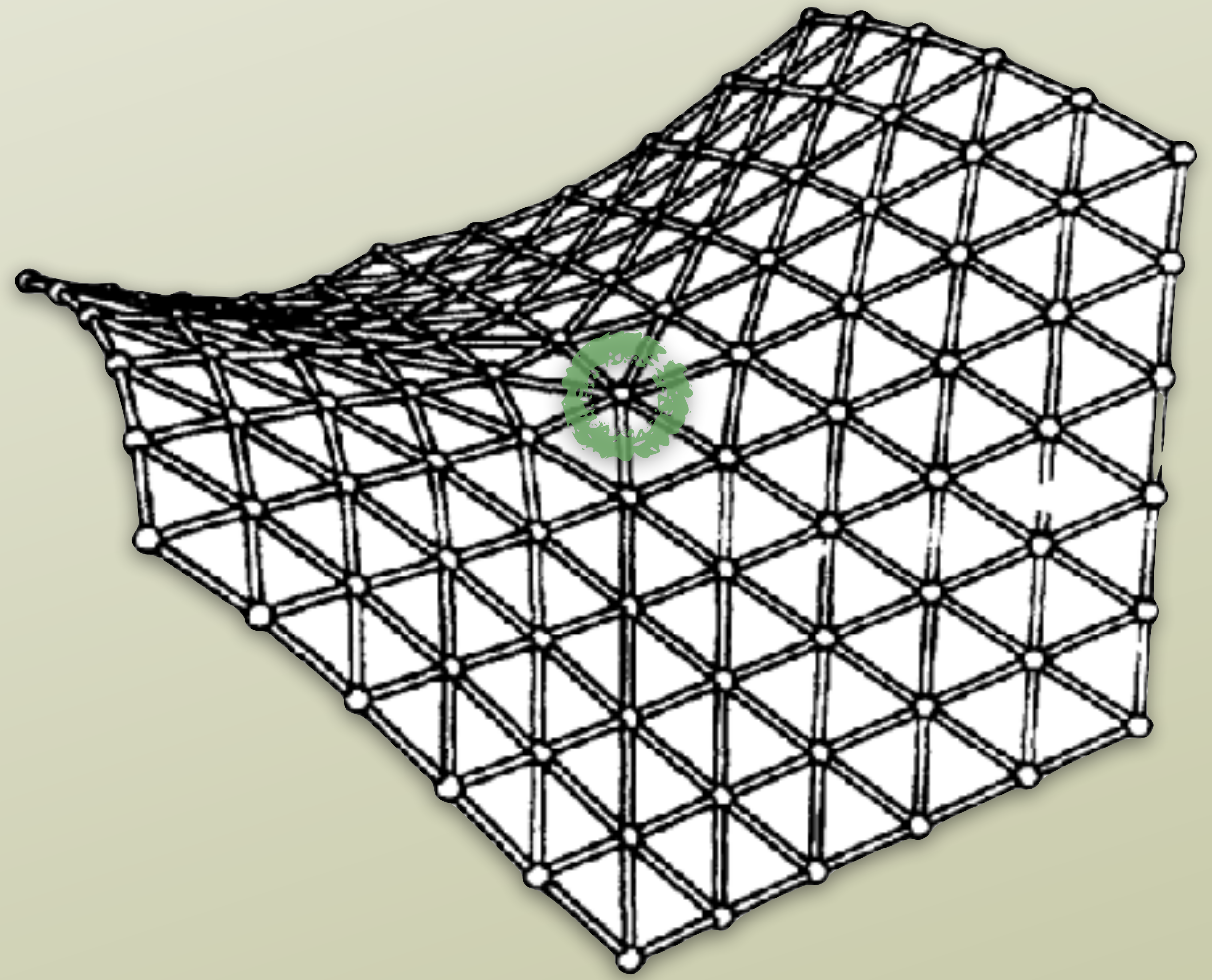

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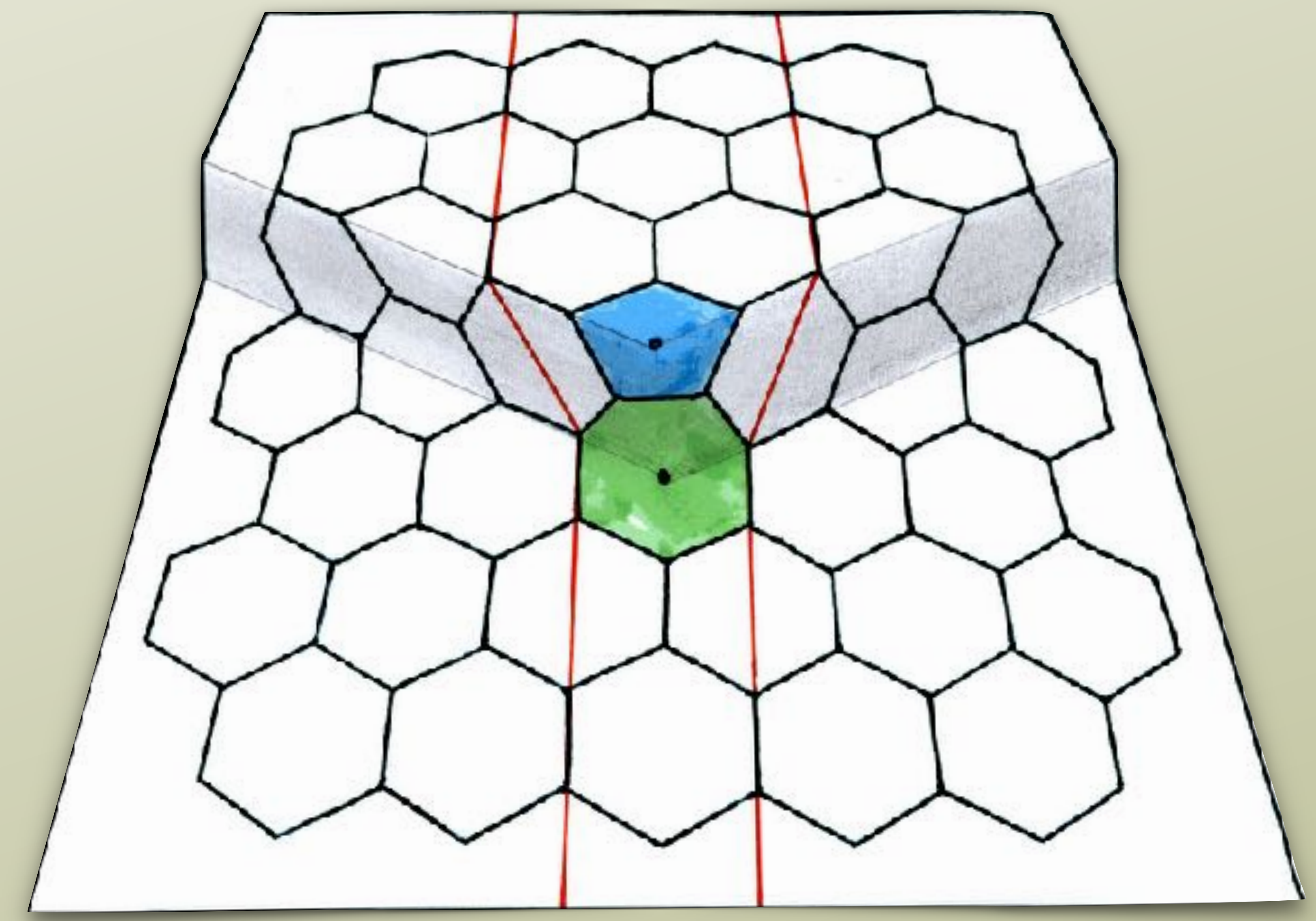
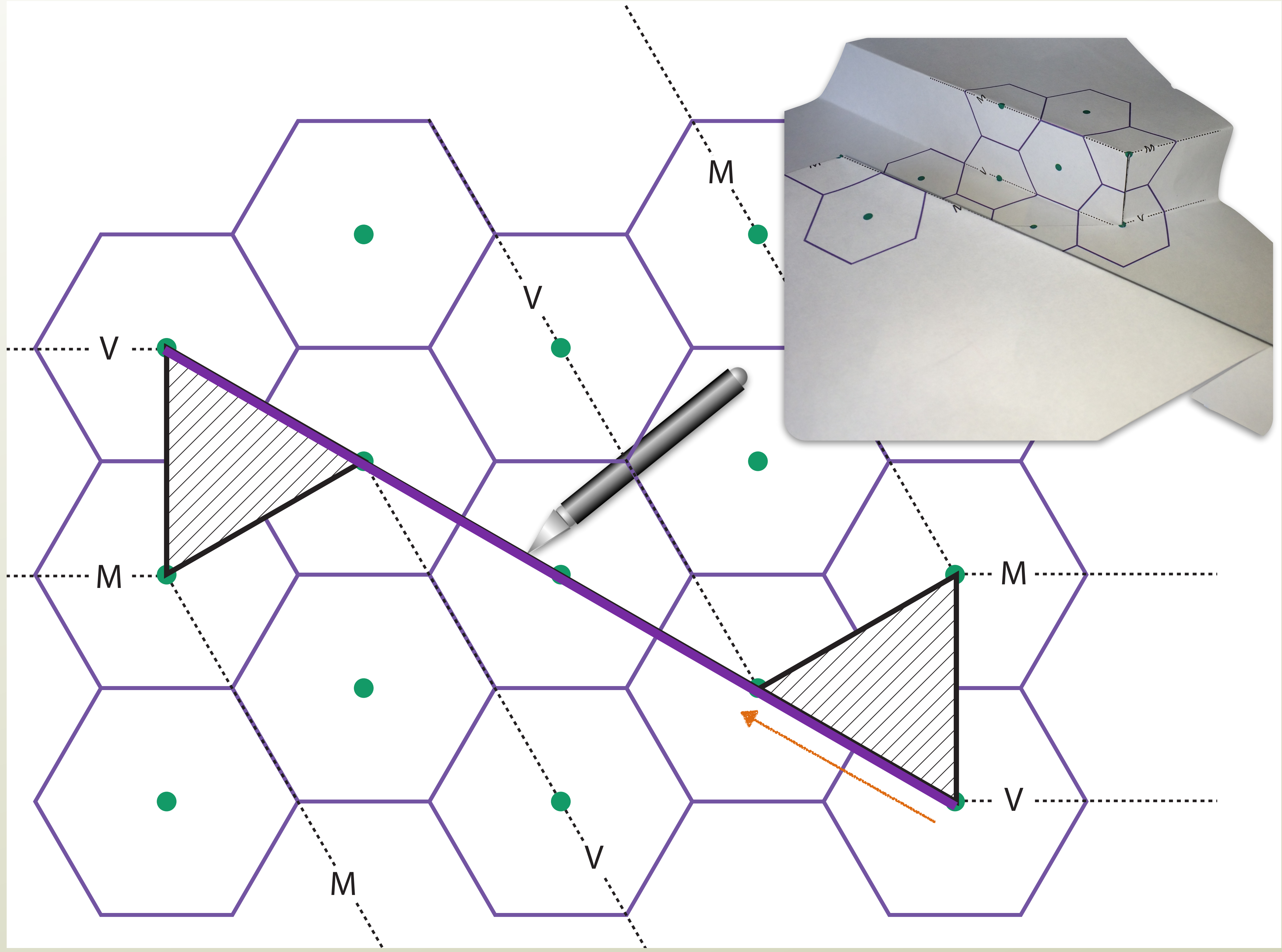
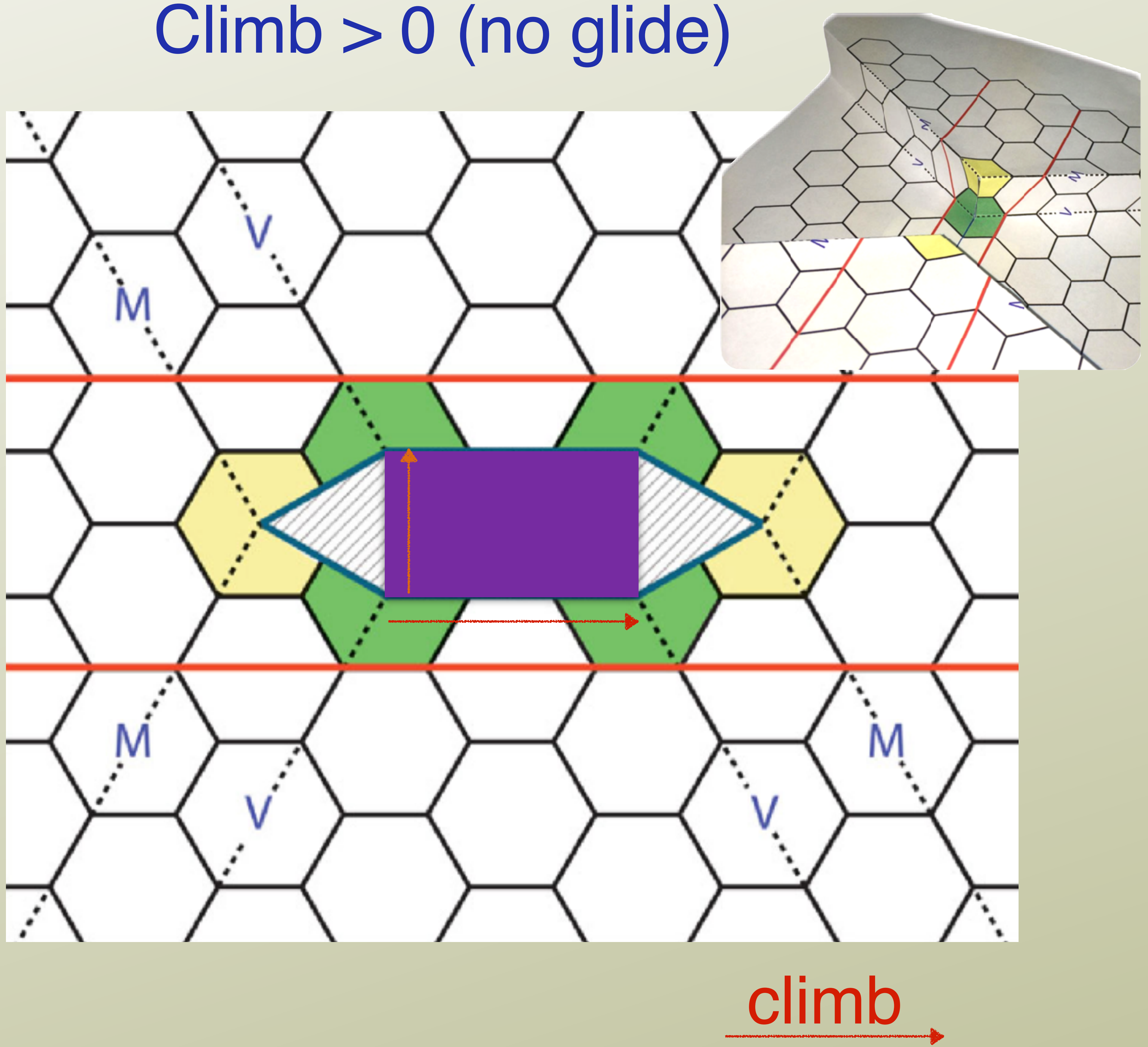


Phyllotaxis: a non conventional crystalline solution to packing efficiency in situations with radial symmetry
J.-F. Sadoc, N. Rivier, and J. Charvolin,
Acta Crystallogr. Sect. A 68, 470 (2012).

Climb = 0 (all glide)



Climb > 0 (no glide)



The amount of material removed corresponds to the area of the wedges (shaded) plus the area of the dislocation (purple).

The area of the dislocation is the product of the Burgers vector and the climb.

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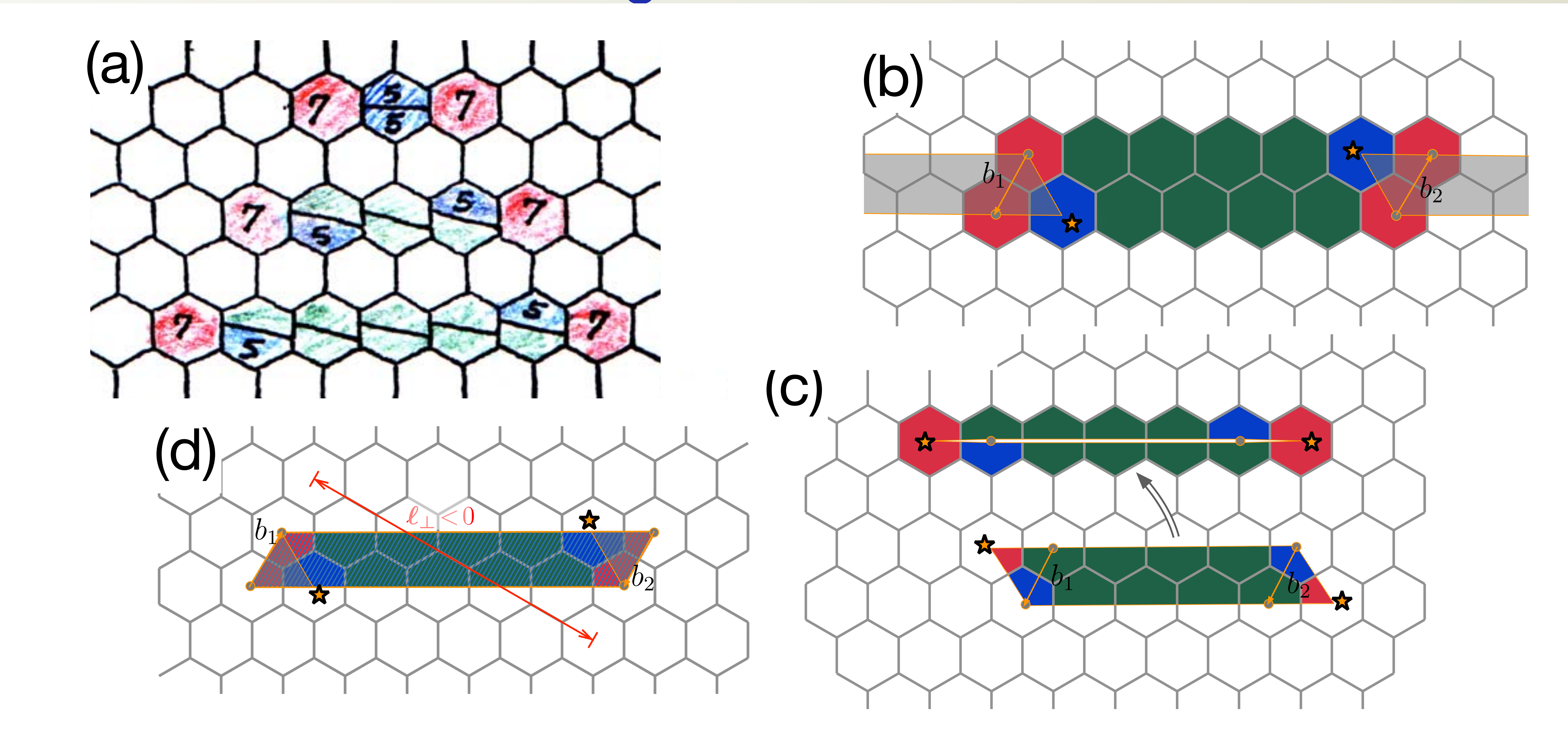
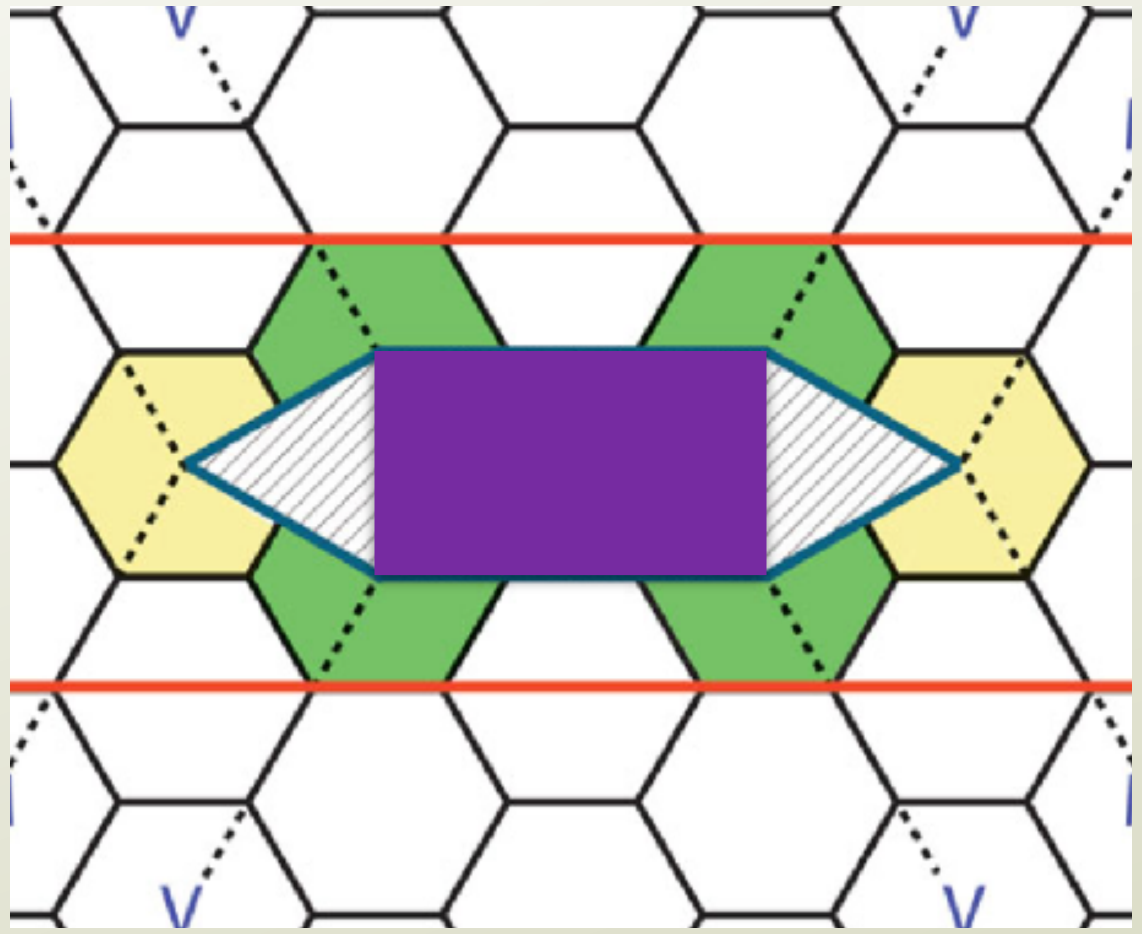
Additive Lattice Kirigami



The language of lattice kirigami can be used to describe a variety of topological changes. If we use *negative* ℓ then we can introduce new material by cutting a slit and glueing extra connected wedges into it.

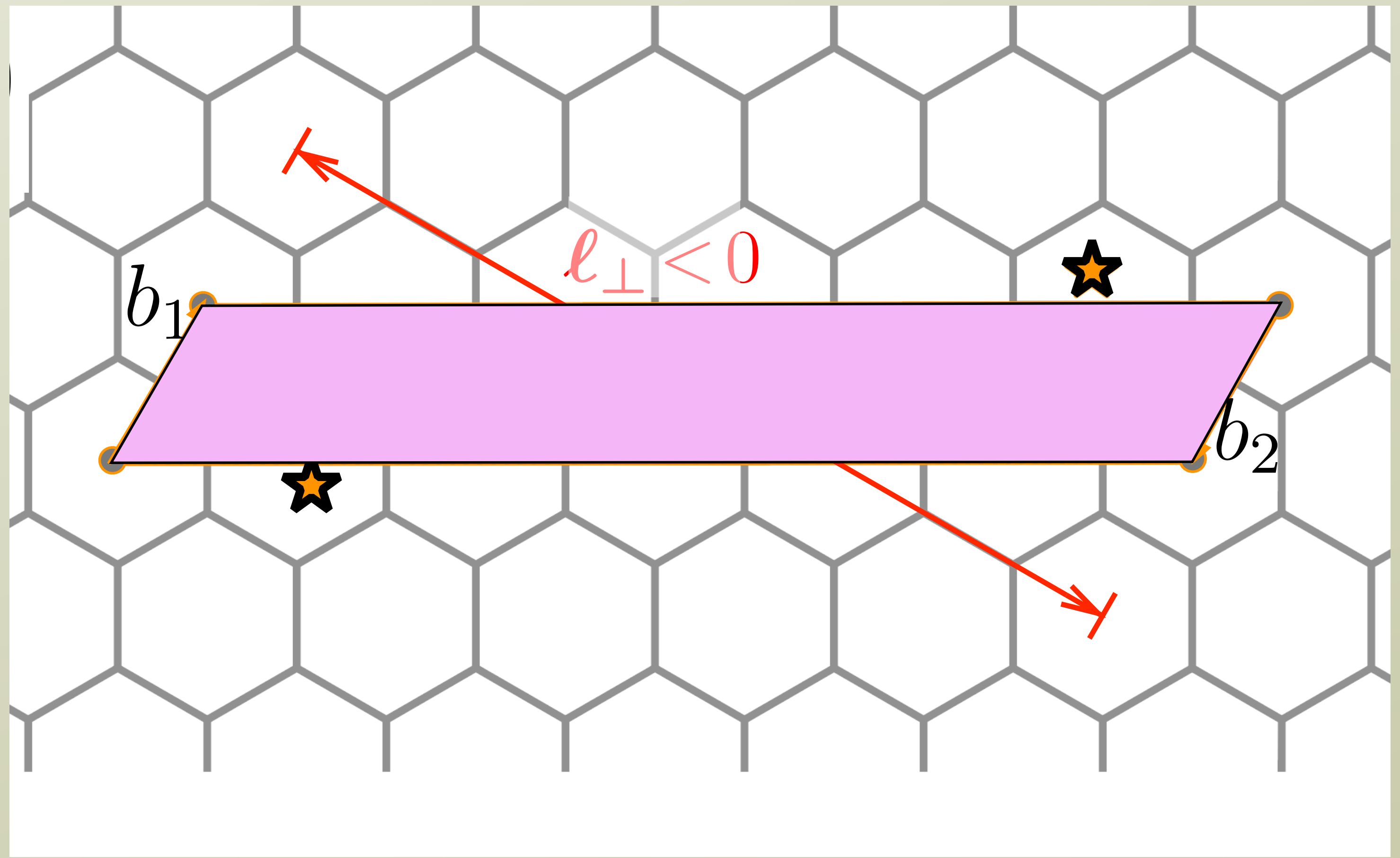
Image (a) from N. Rivier, M. F. Miri, C. Oguey, Plasticity and topological defects in cellular structures: Extra matter, folds and crab moulting. Colloids Surf. A 263, 39–45 (2005).

Material lost or gained in the dislocation

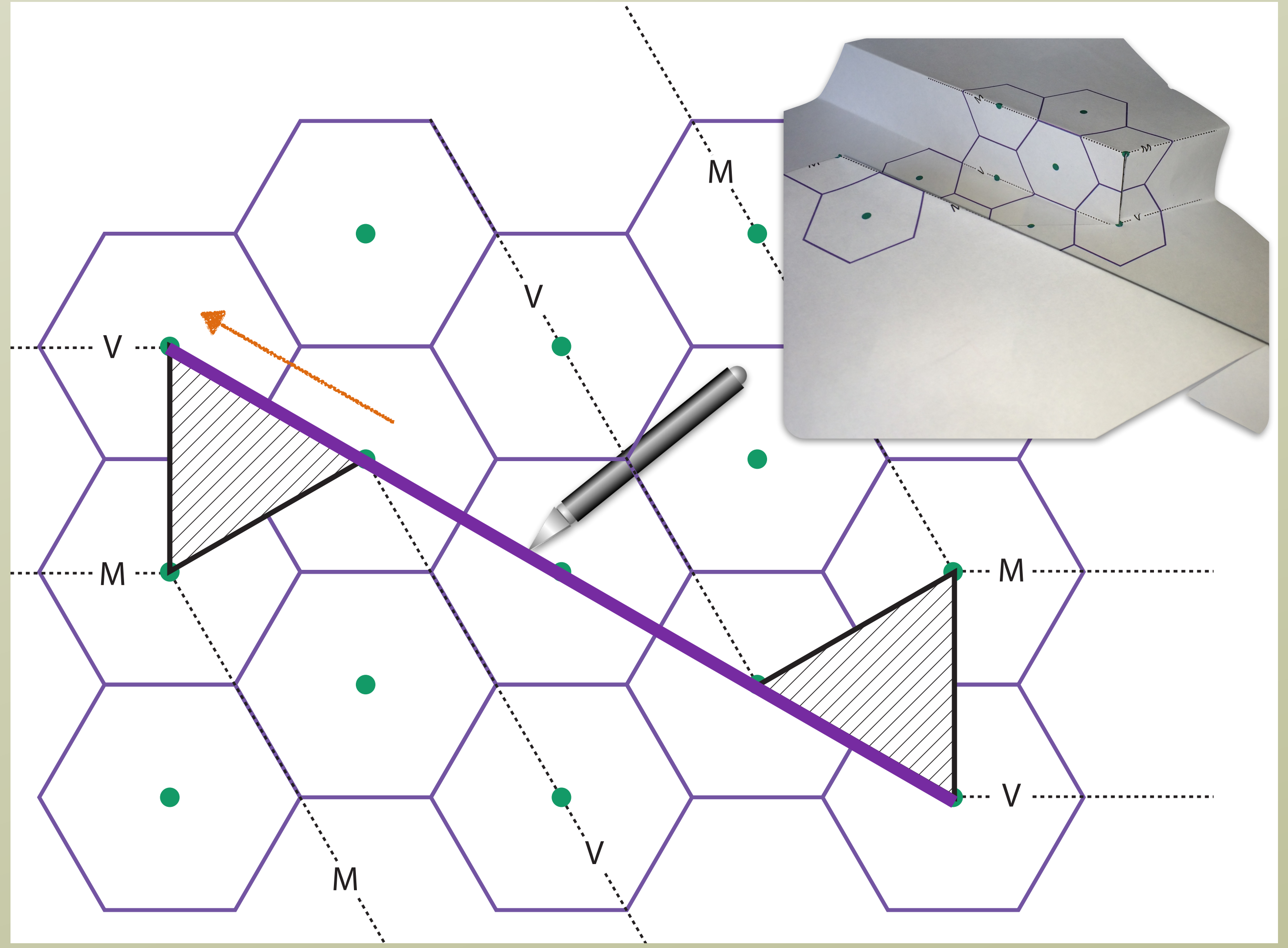


material loss

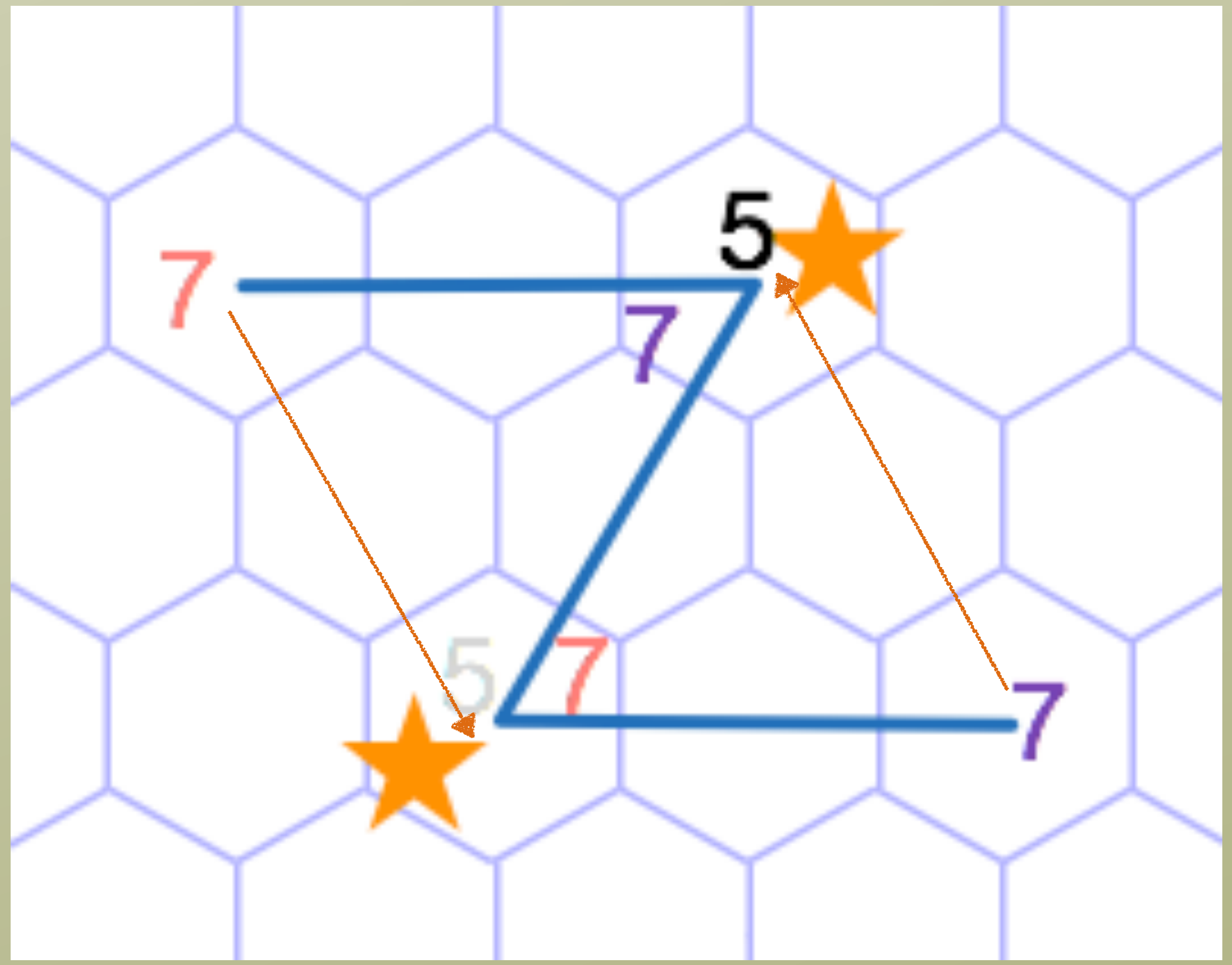
material gain



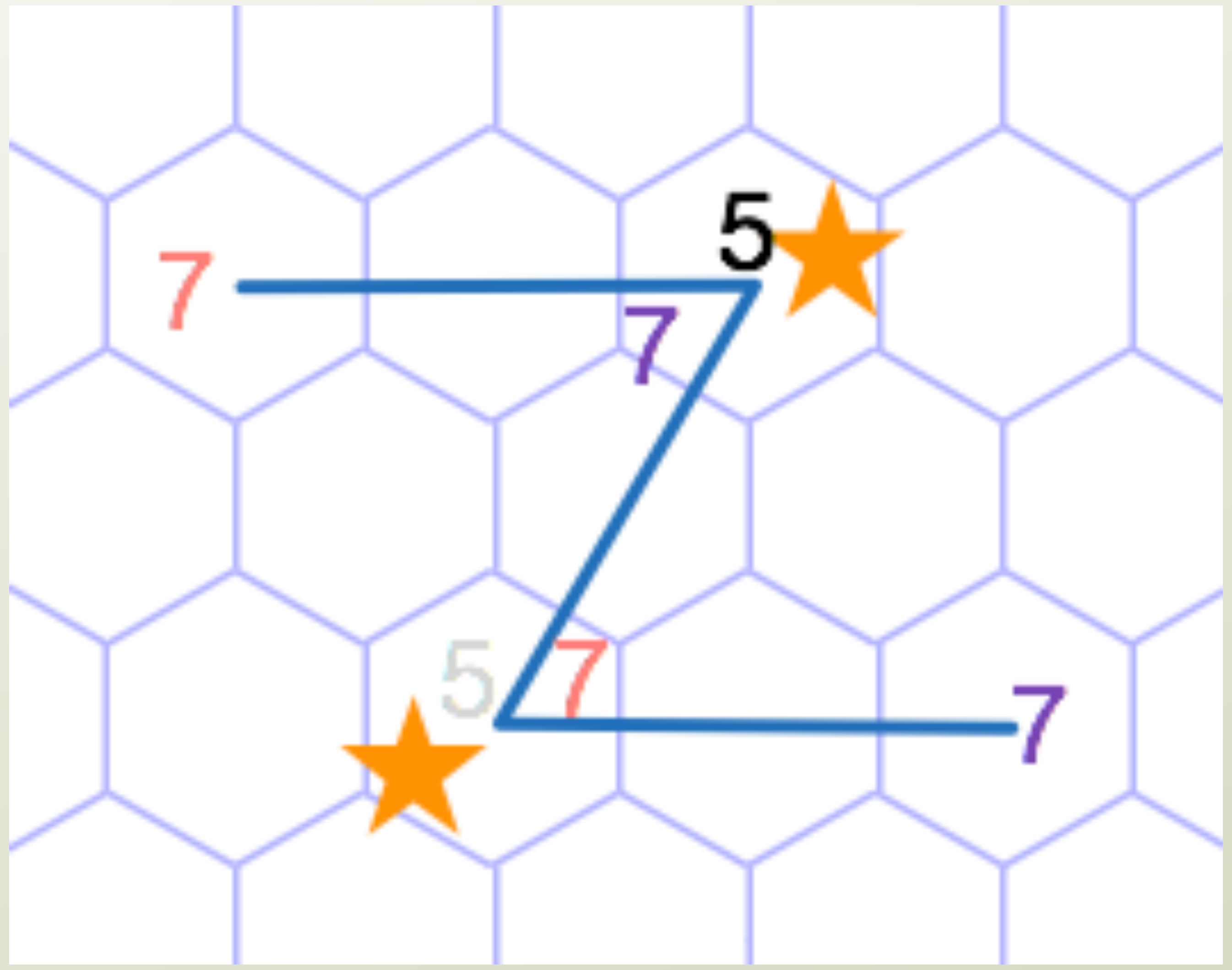
no loss through dislocation

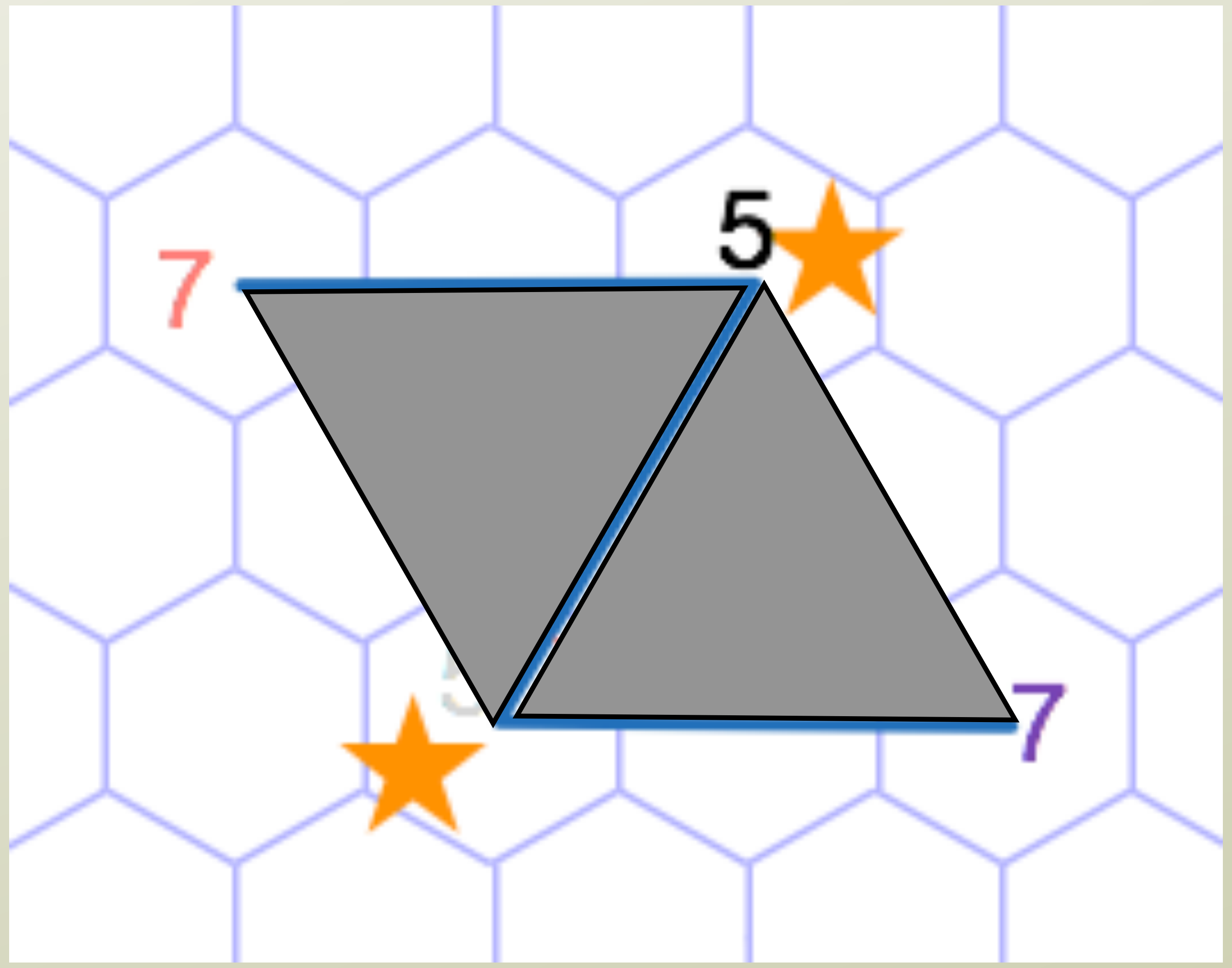


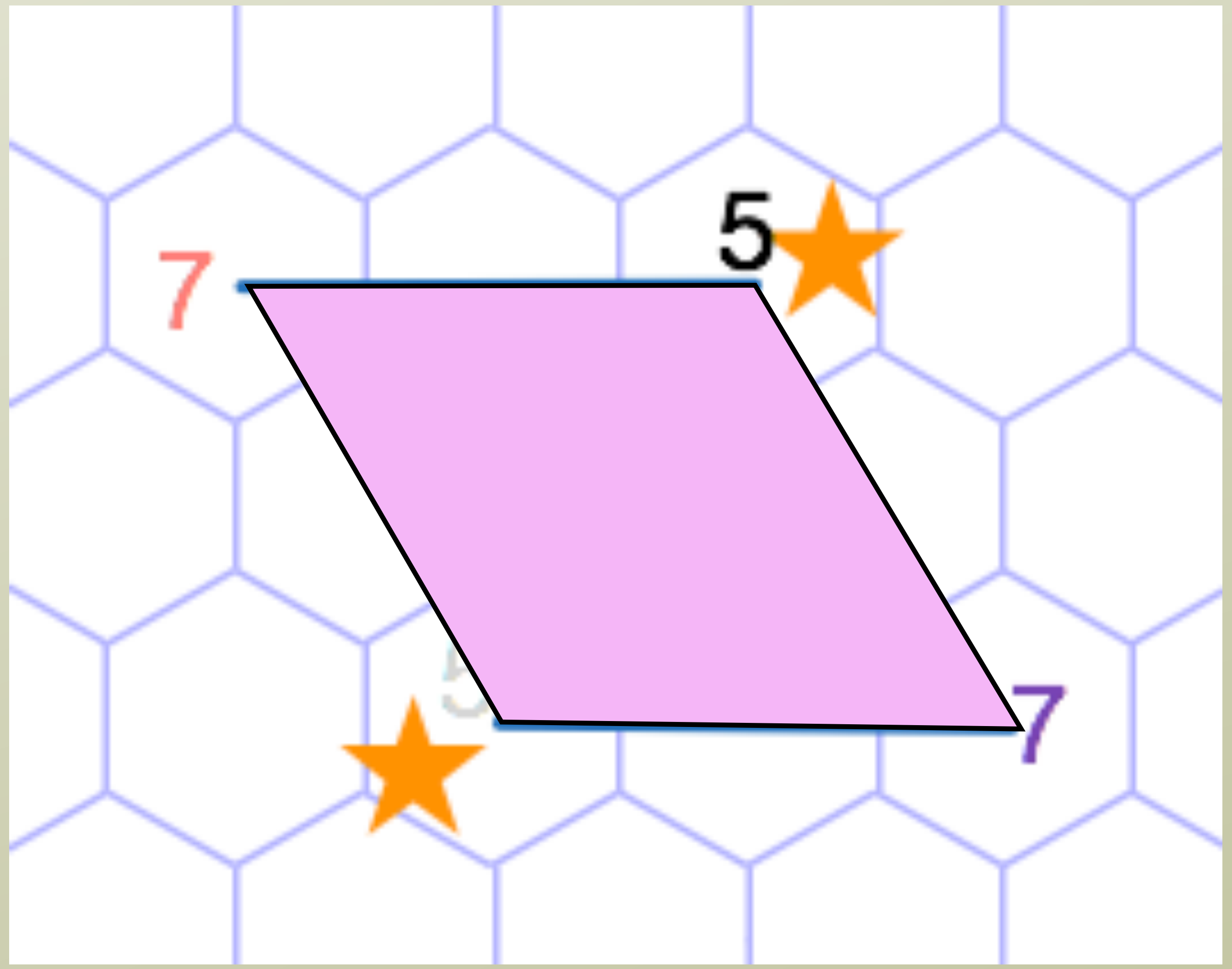
gain through dislocation = loss from wedges



Material neutral cuts





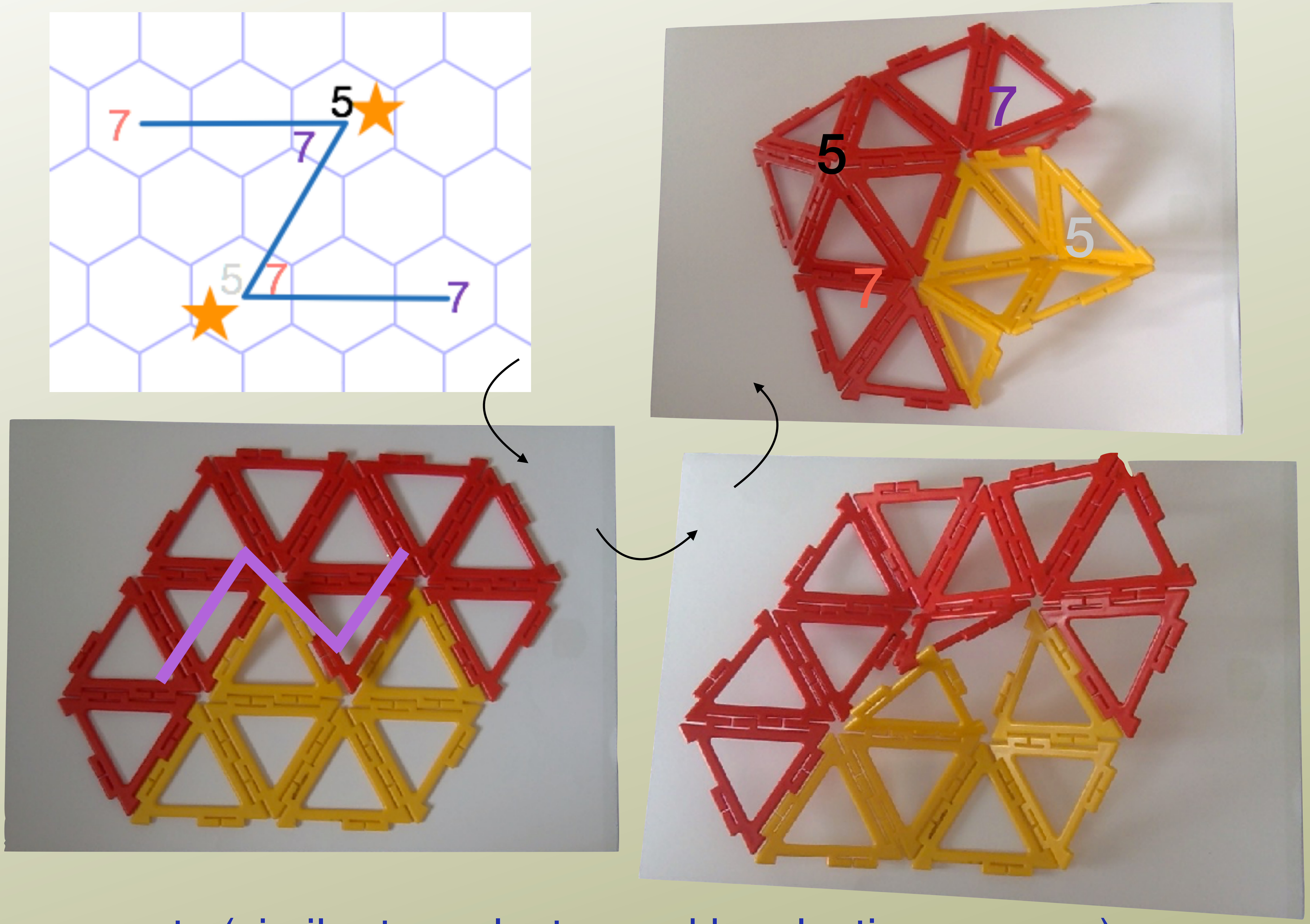


Material lost (wedges)

=

Material gained (negative climb)

Material neutral cuts



z cuts (similar to z-plasty used by plastic surgeons)

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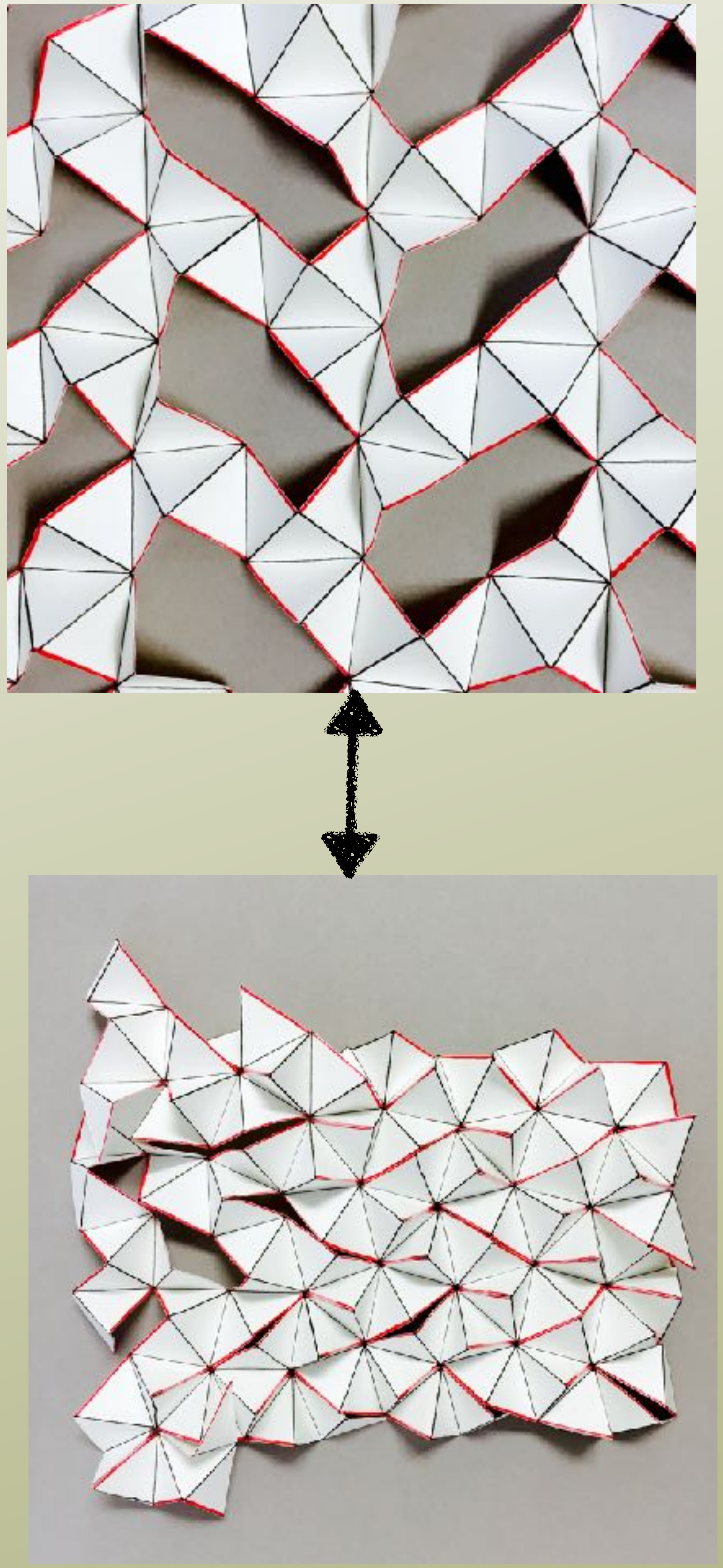
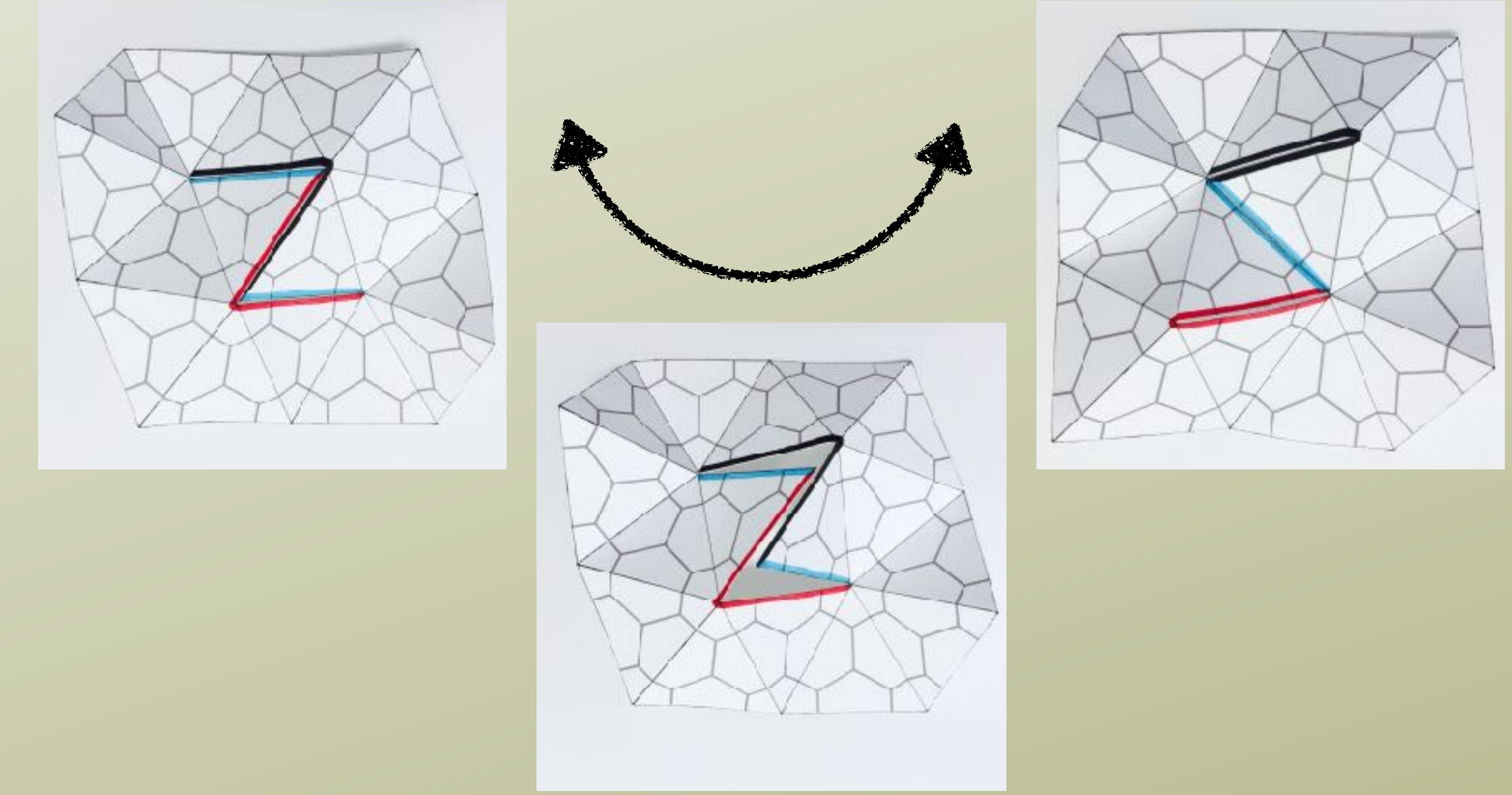
General result:

- *any* cut made in the lattice,
- followed by *any* consistent addition or removal of material,
- followed by *any* manner of rejoining the cut edges

is (additive lattice) kirigami





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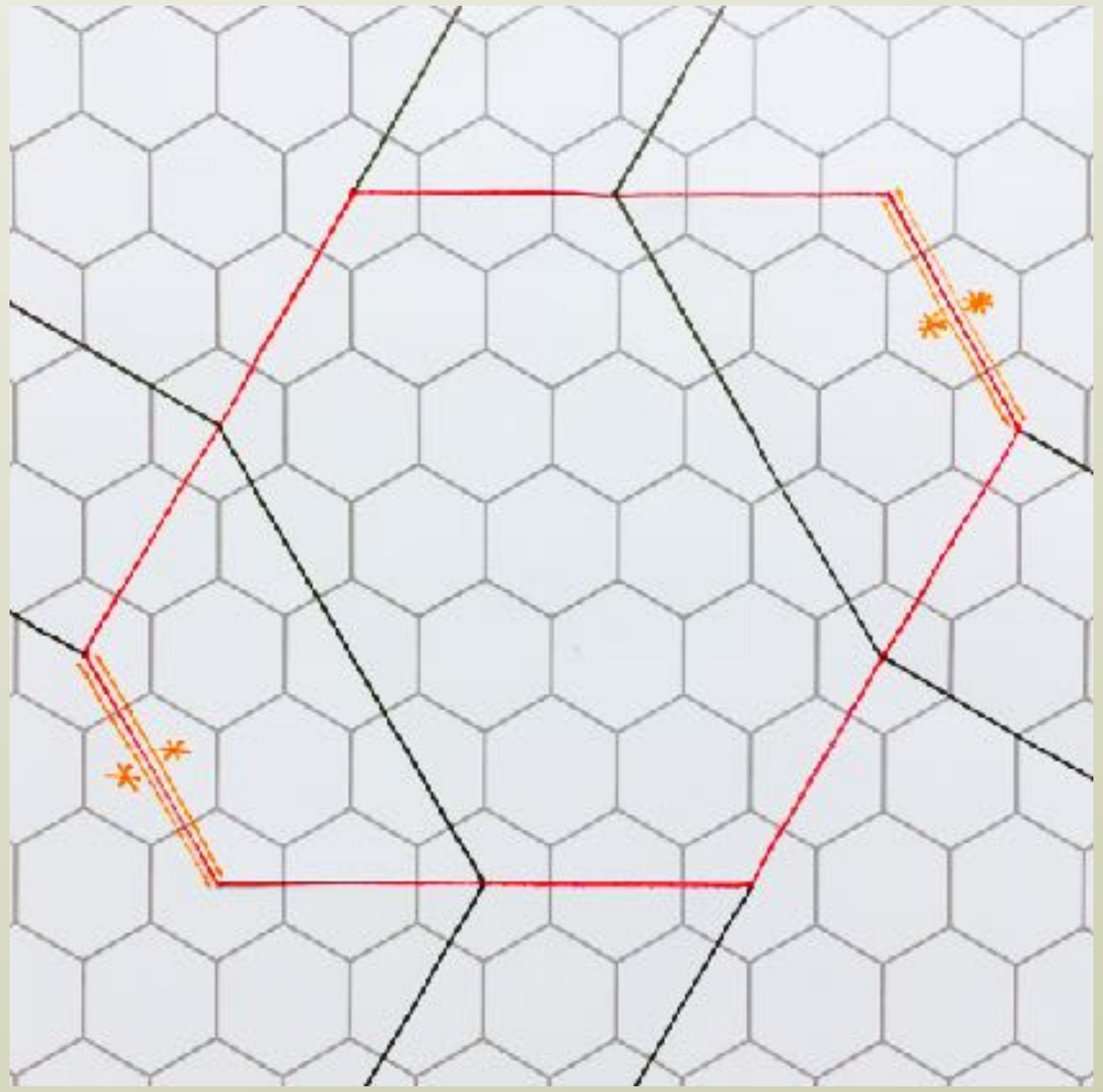
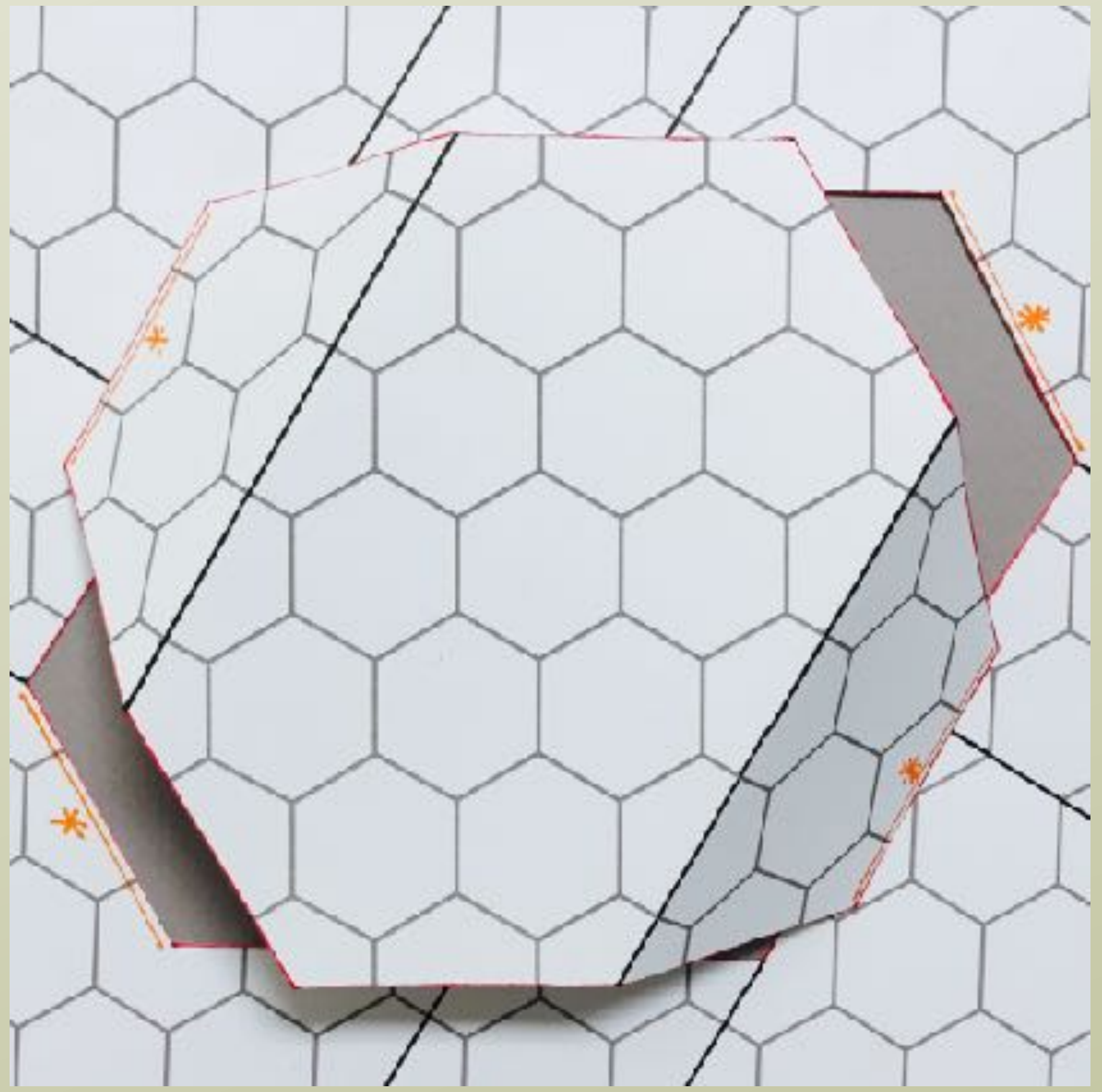
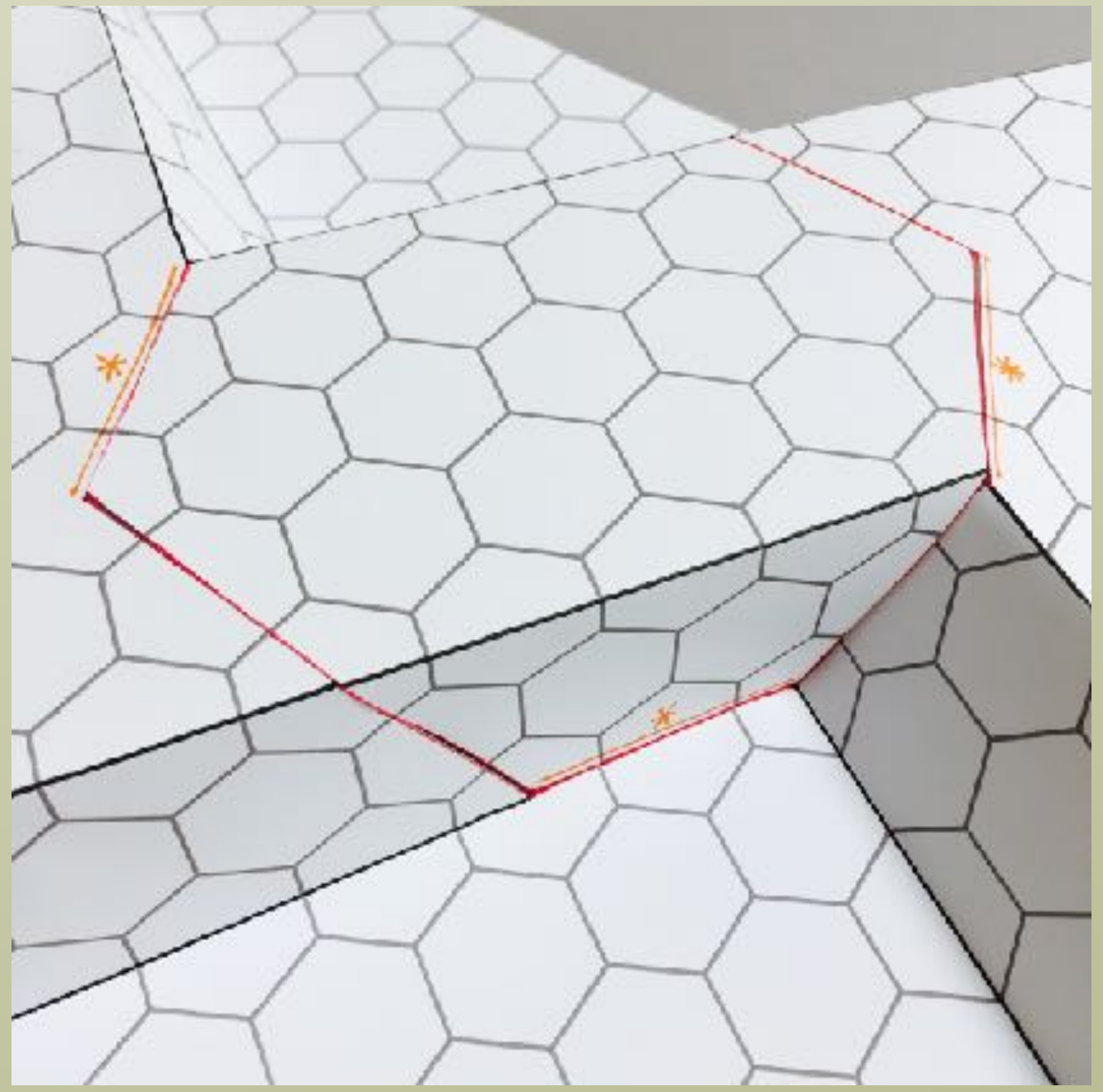
Combining additive and subtractive kirigami elements



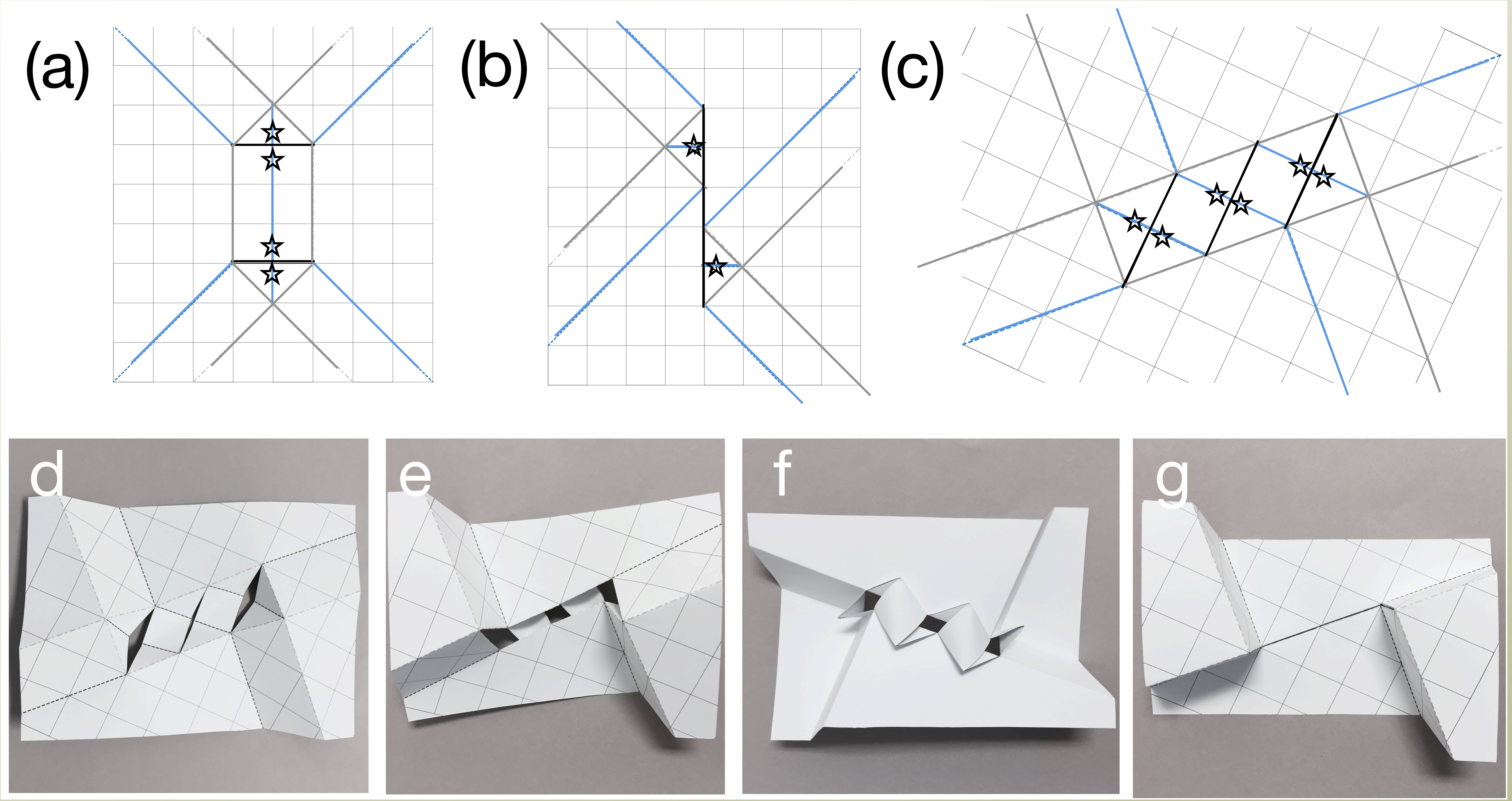


subtractive



additive

Slit-cut kirigami



follows the rules of reglueing boundaries

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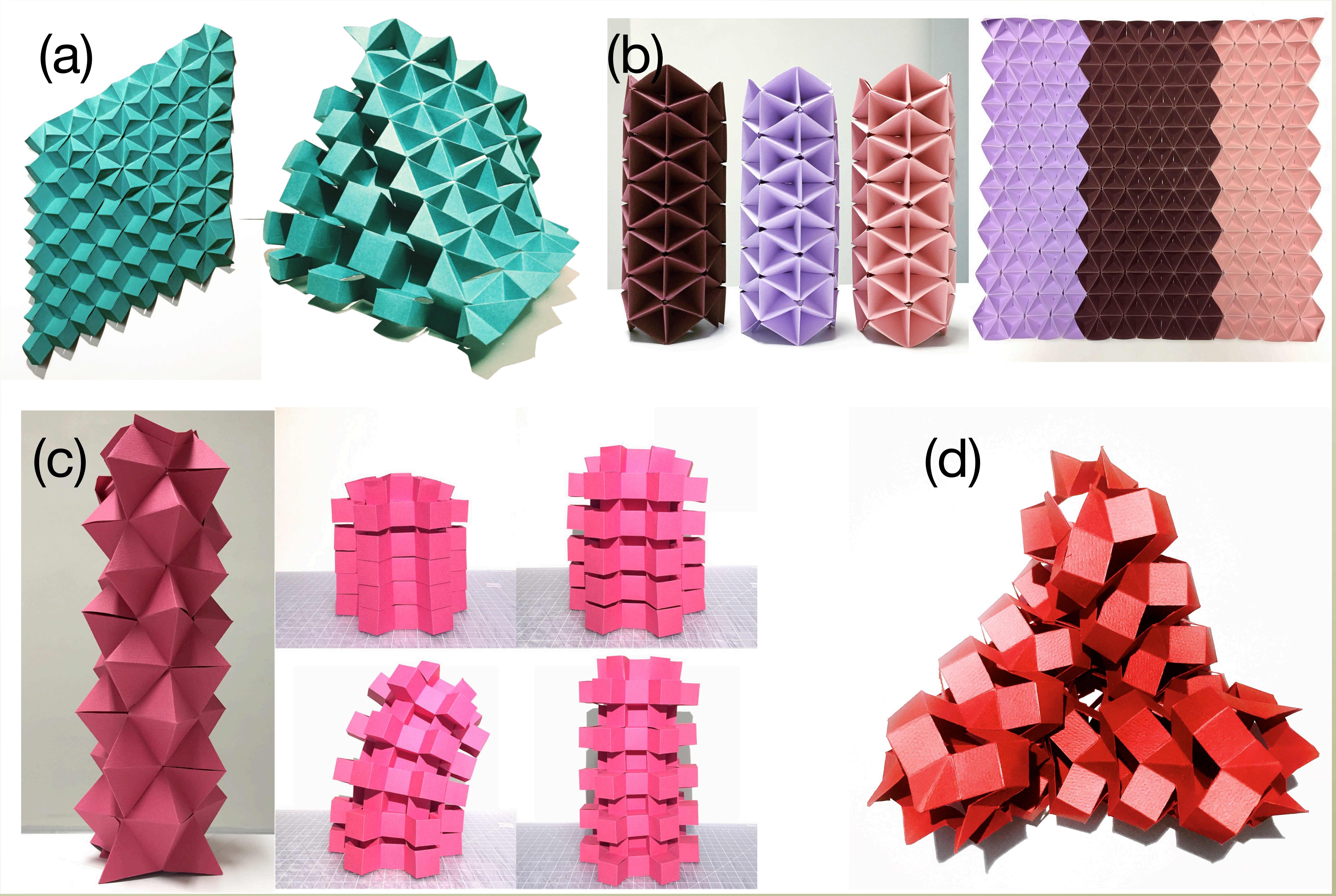

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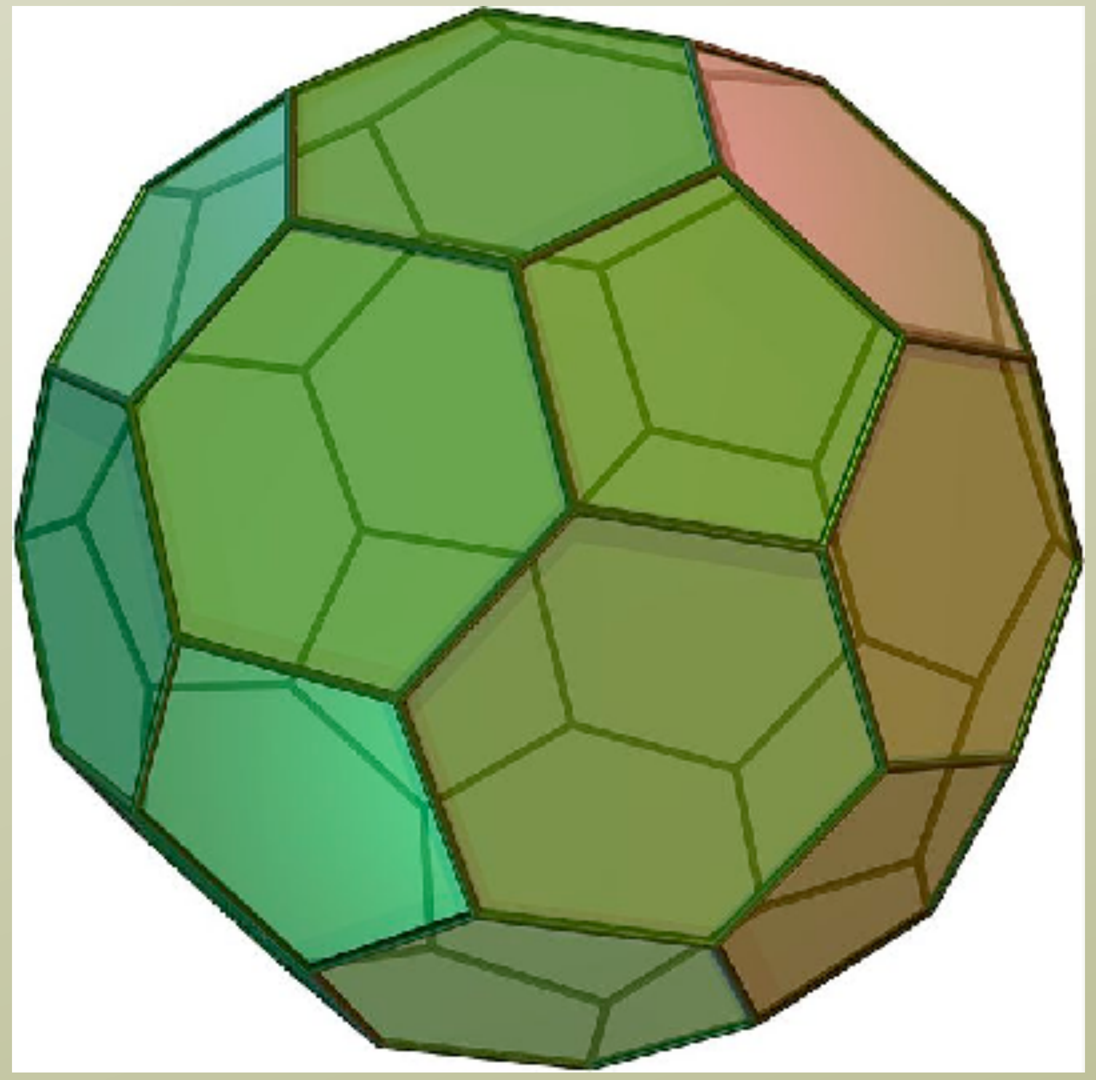
Creating curvature by glueing together mismatched shapes

The dream

The reality



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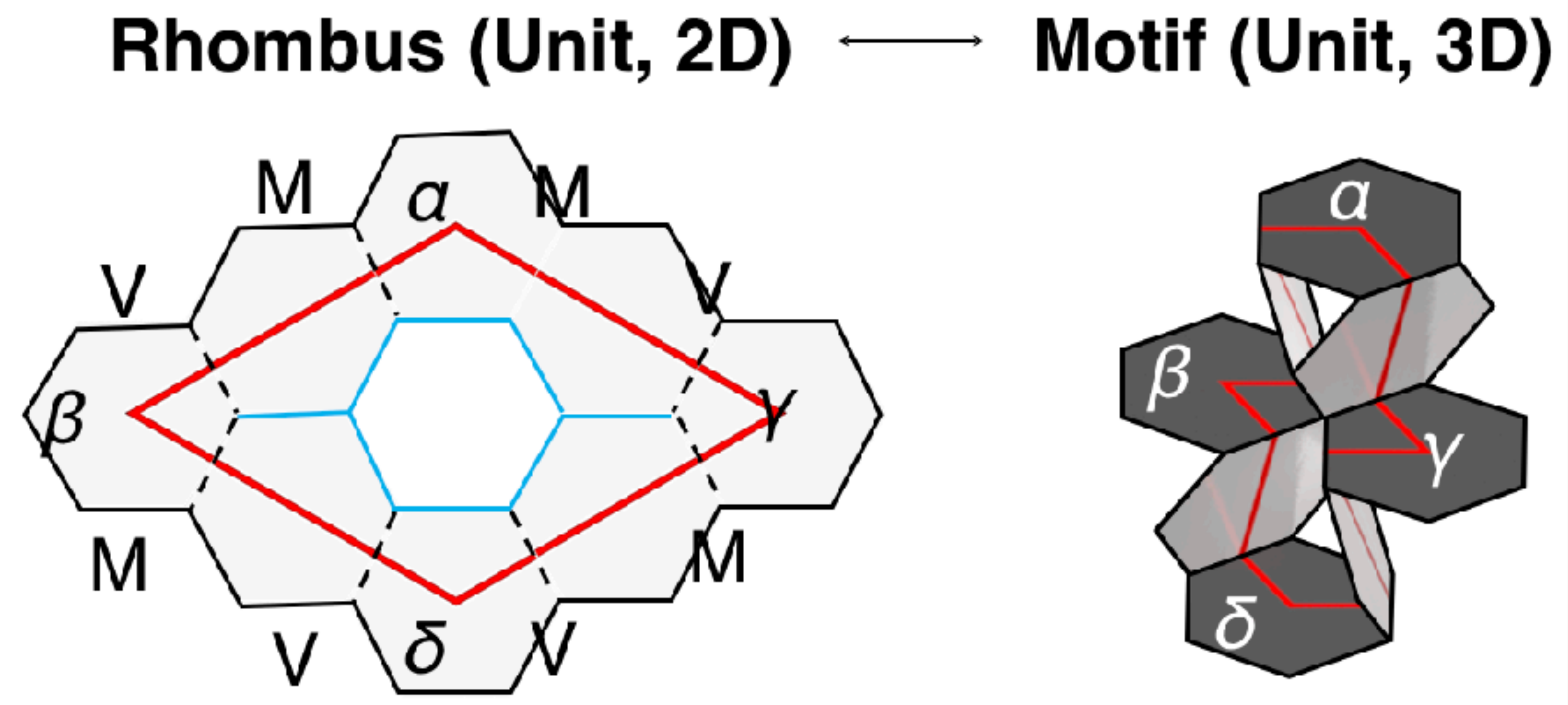
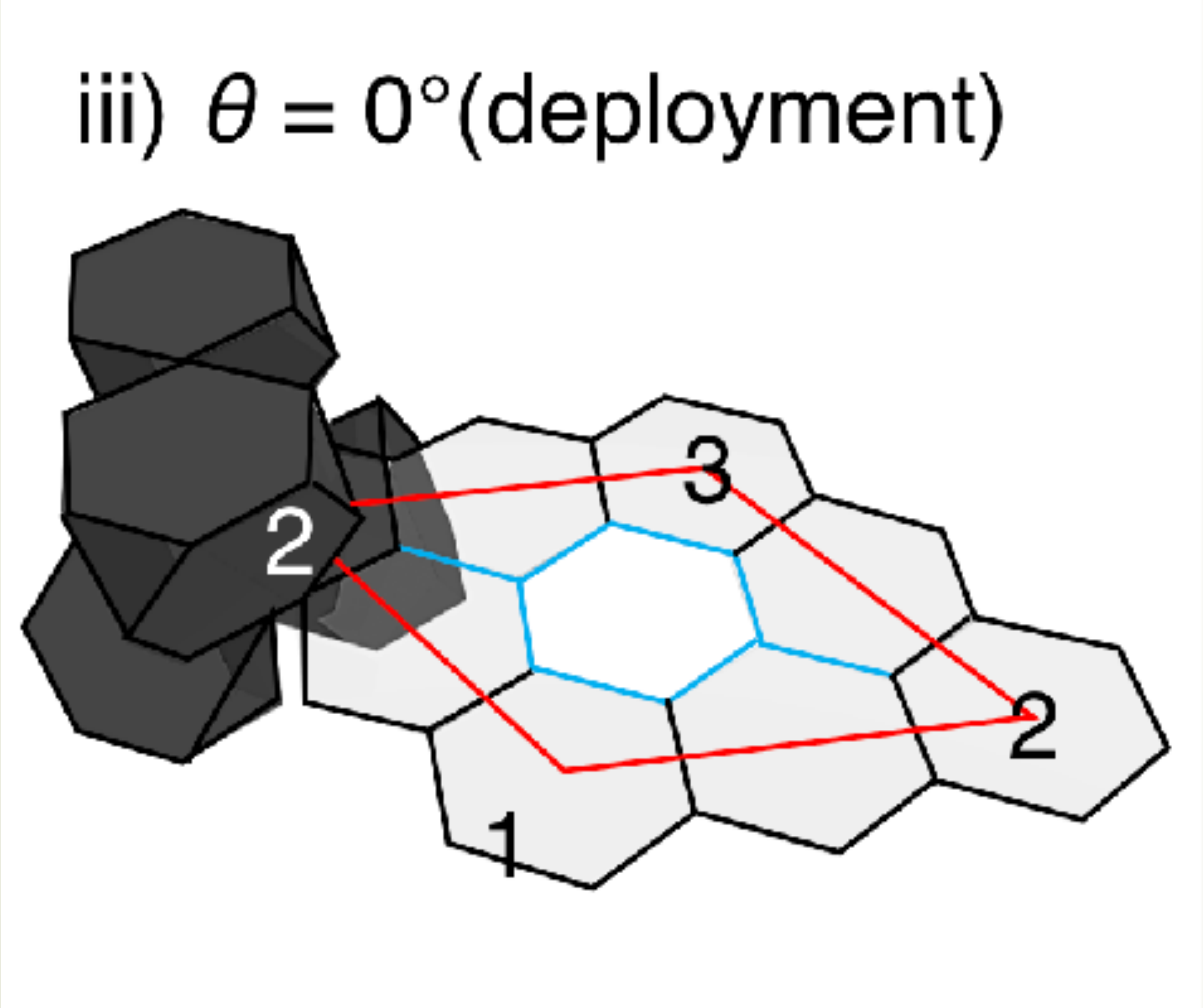
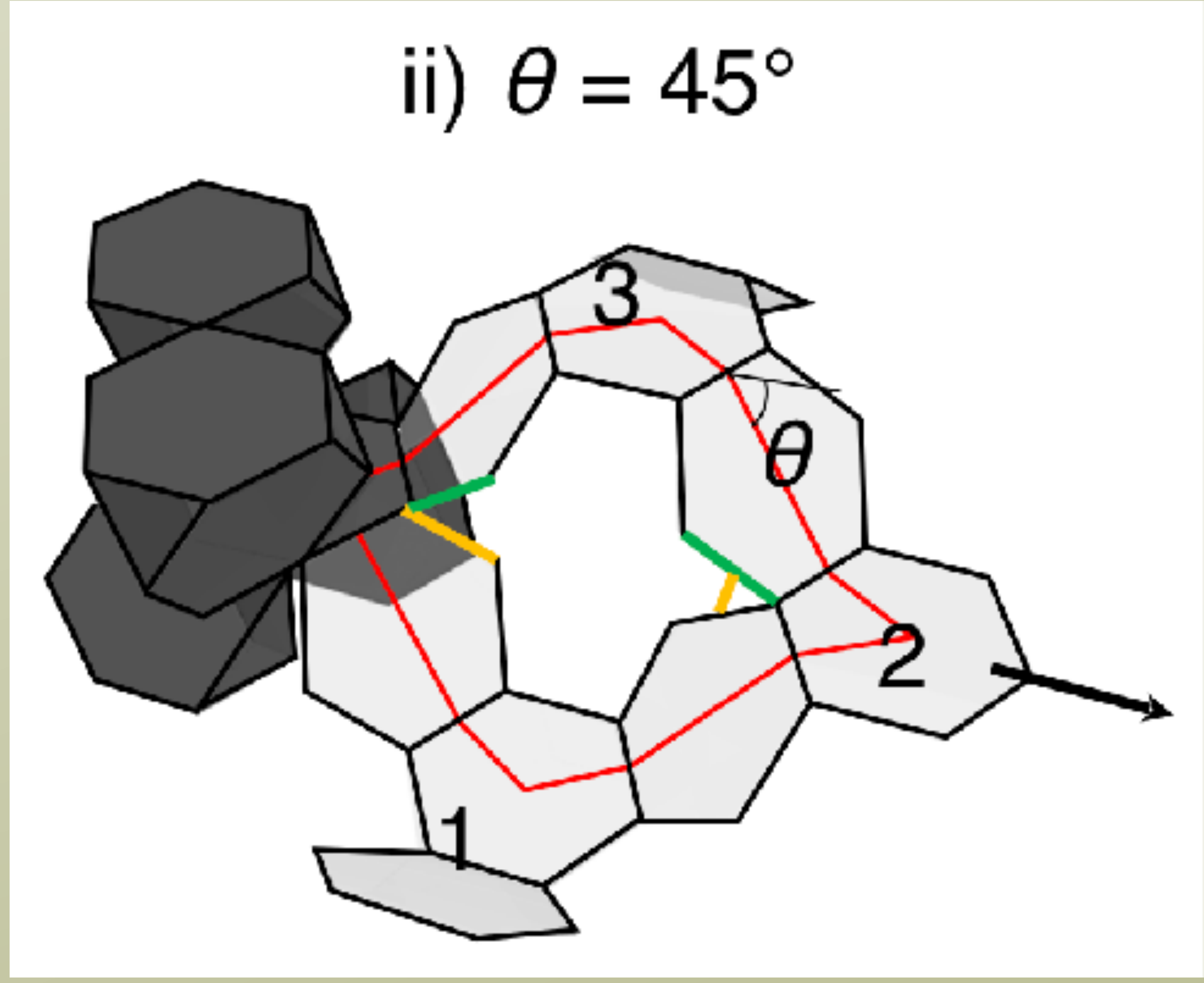
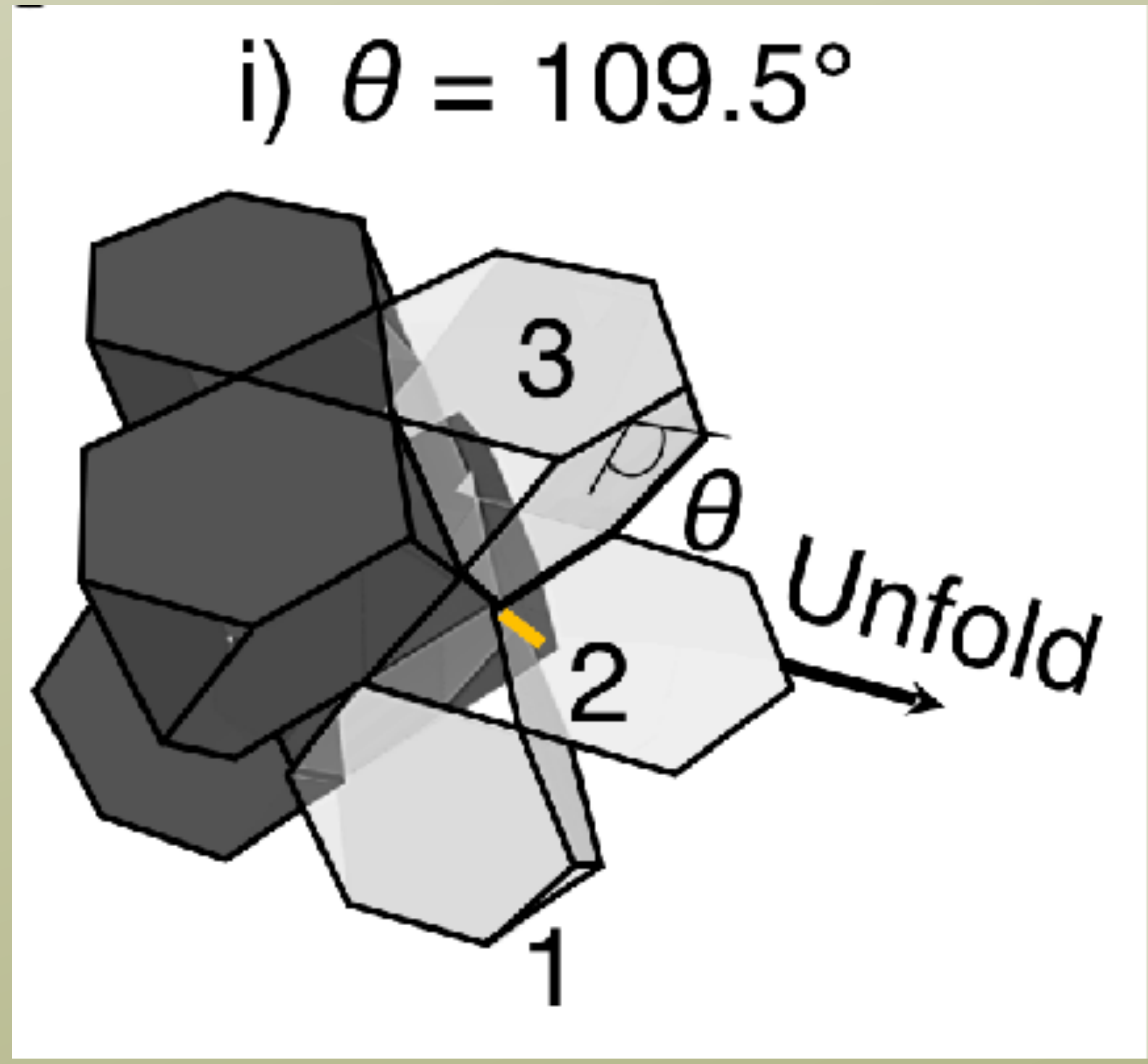
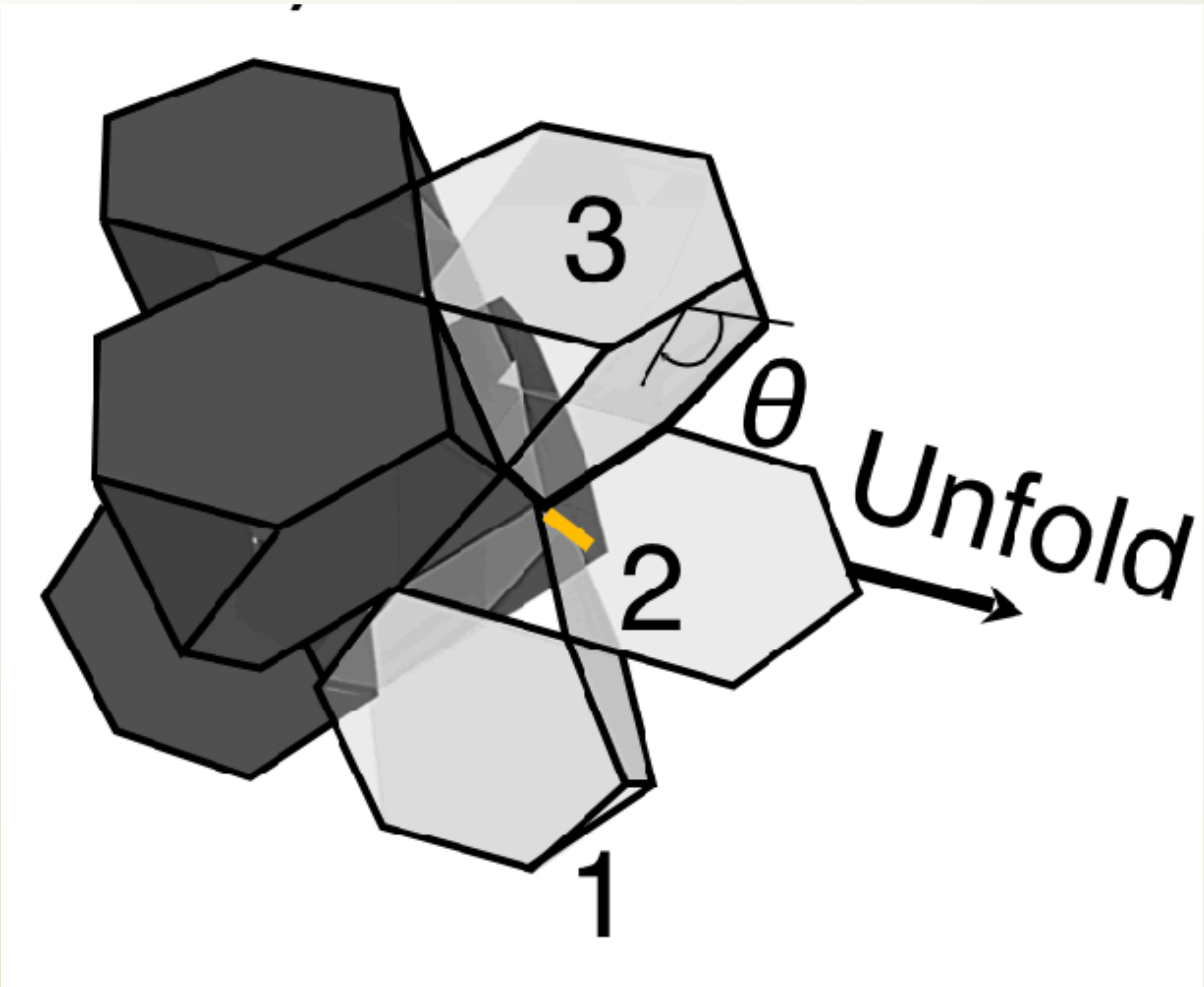
Flat plane to high genus







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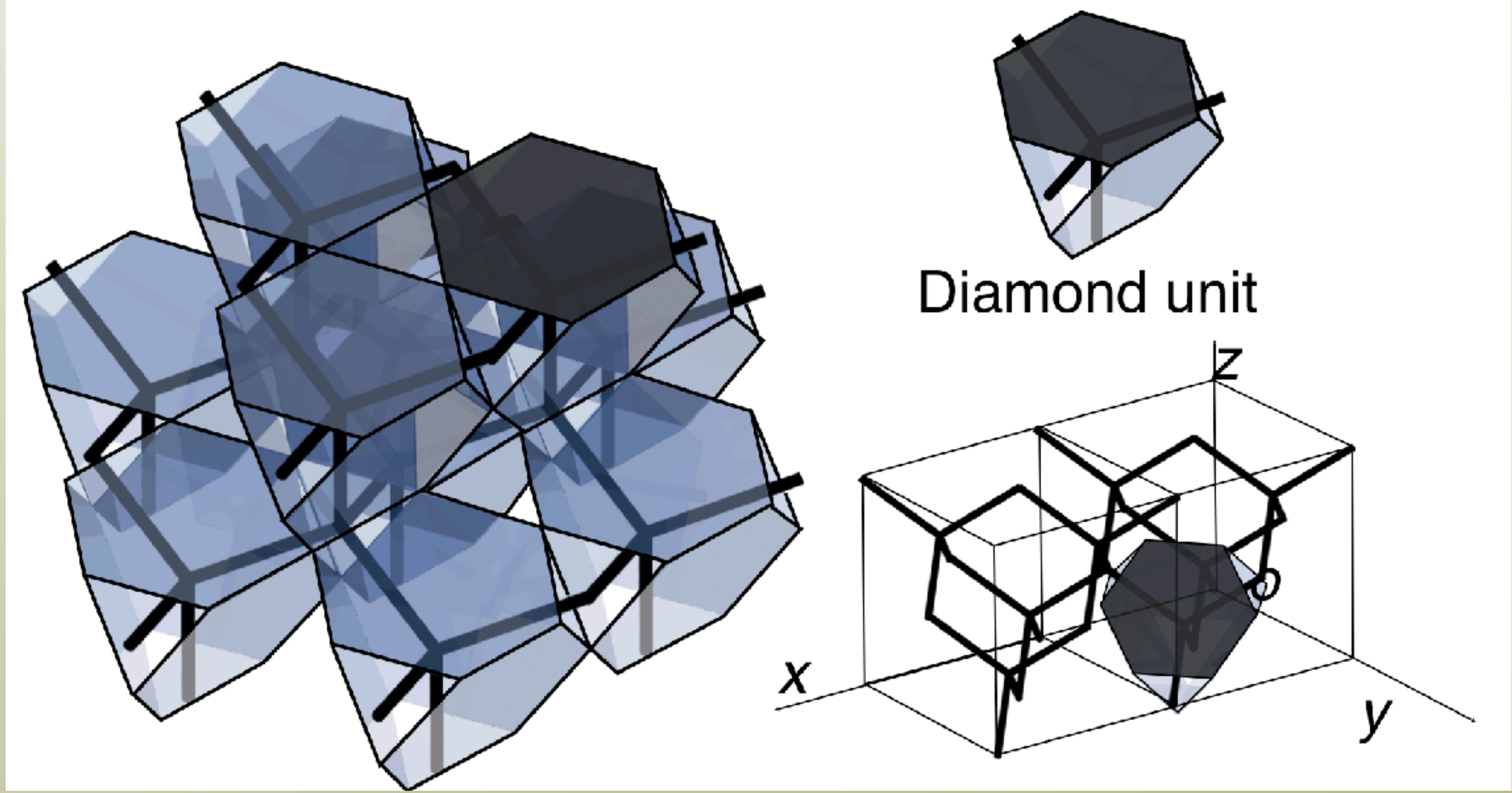
Diamond unit
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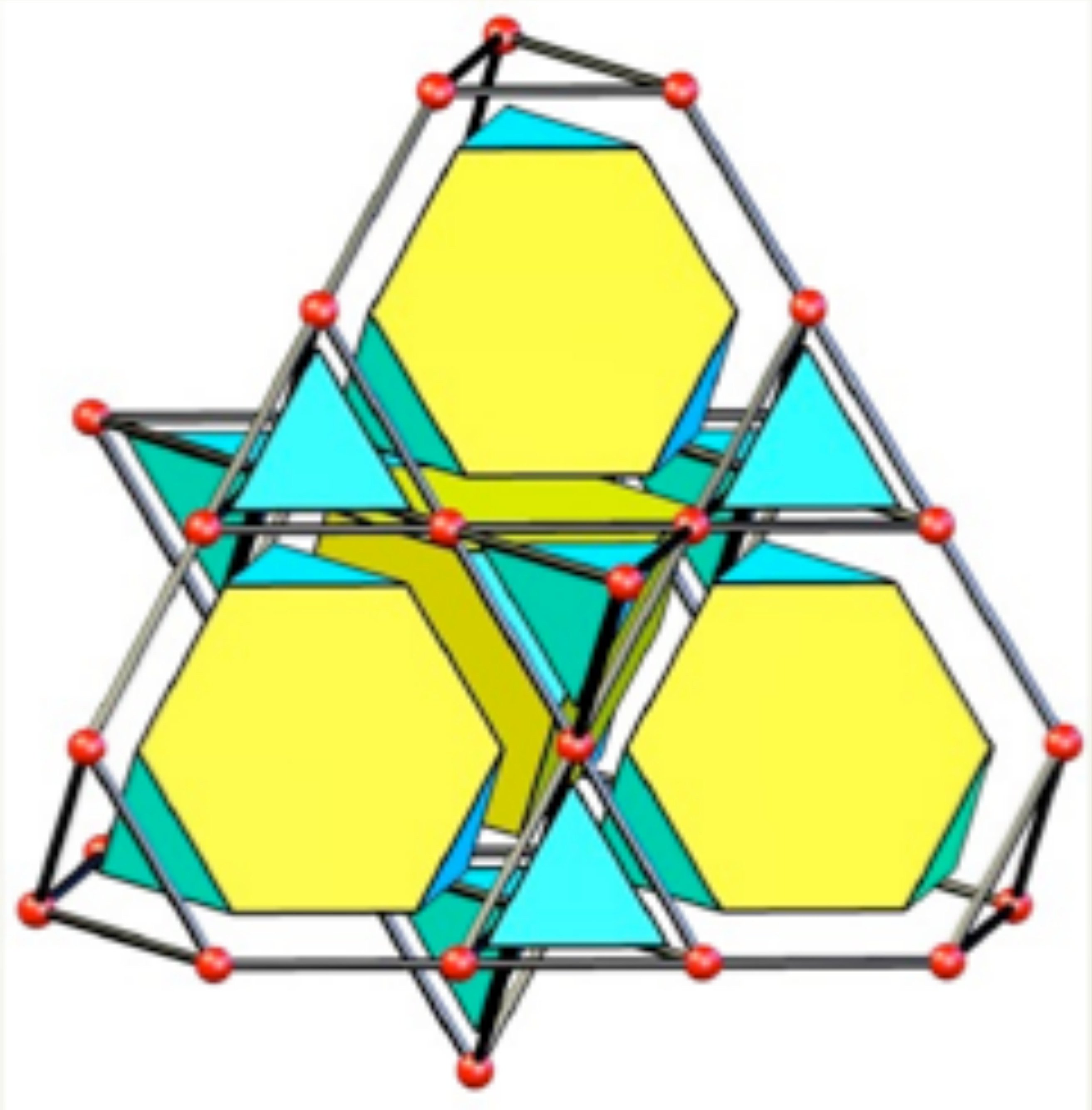
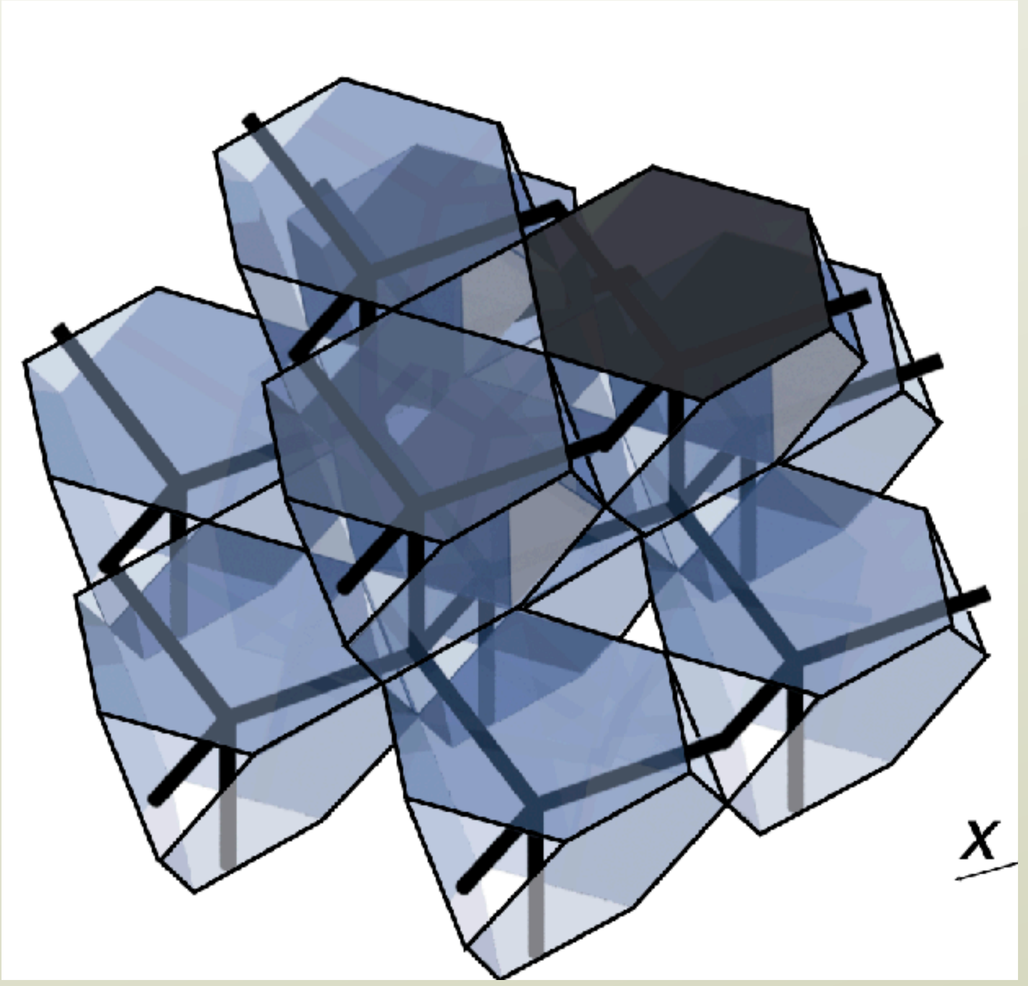
cristobalite/pyrochlore



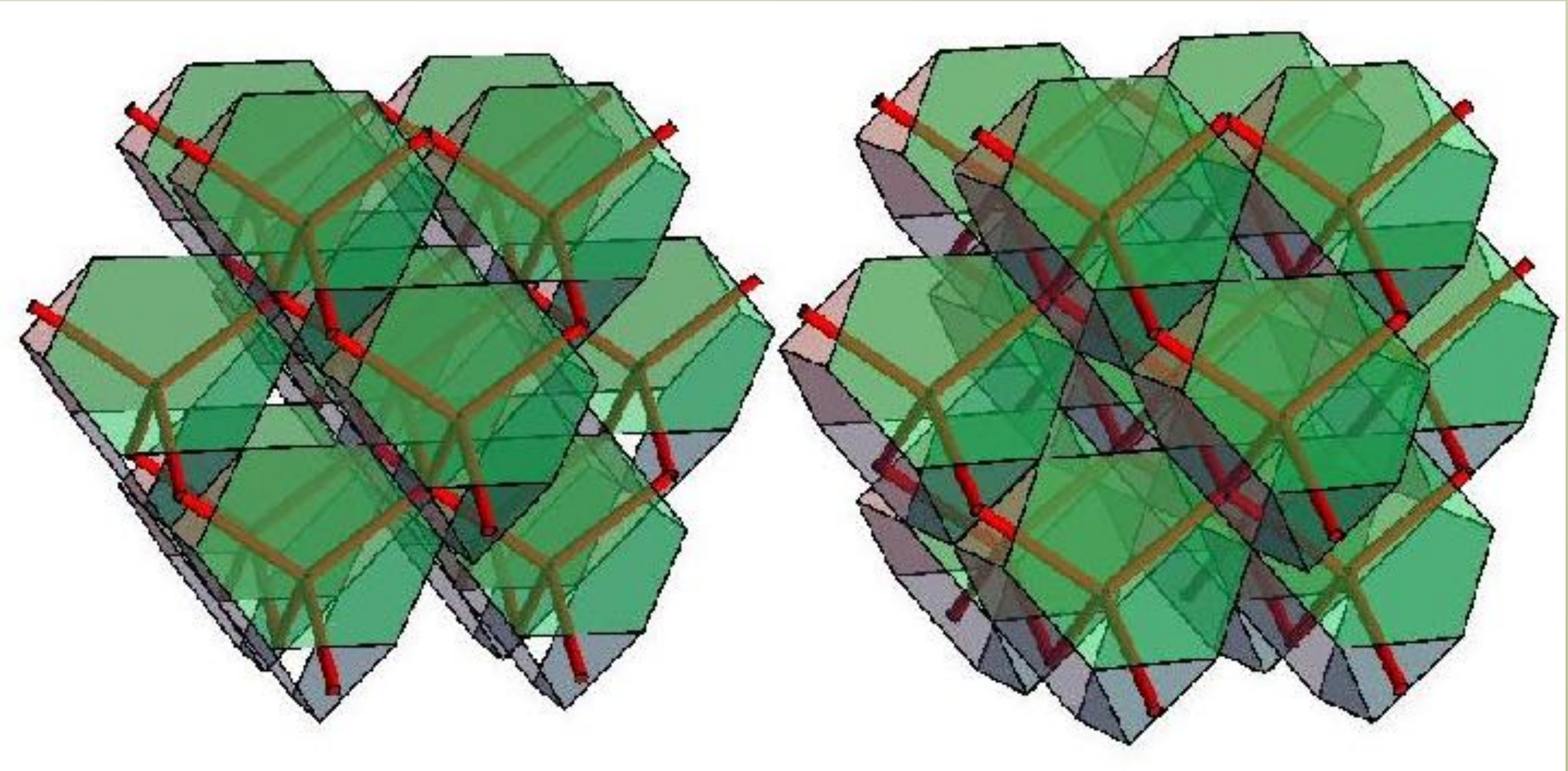
from RCSR



ours

{6,6} tiling on the D surface

from *epinet*



Coxeter-Petrie regular skew polyhedron
{6,6|3} from schoengeometry.com

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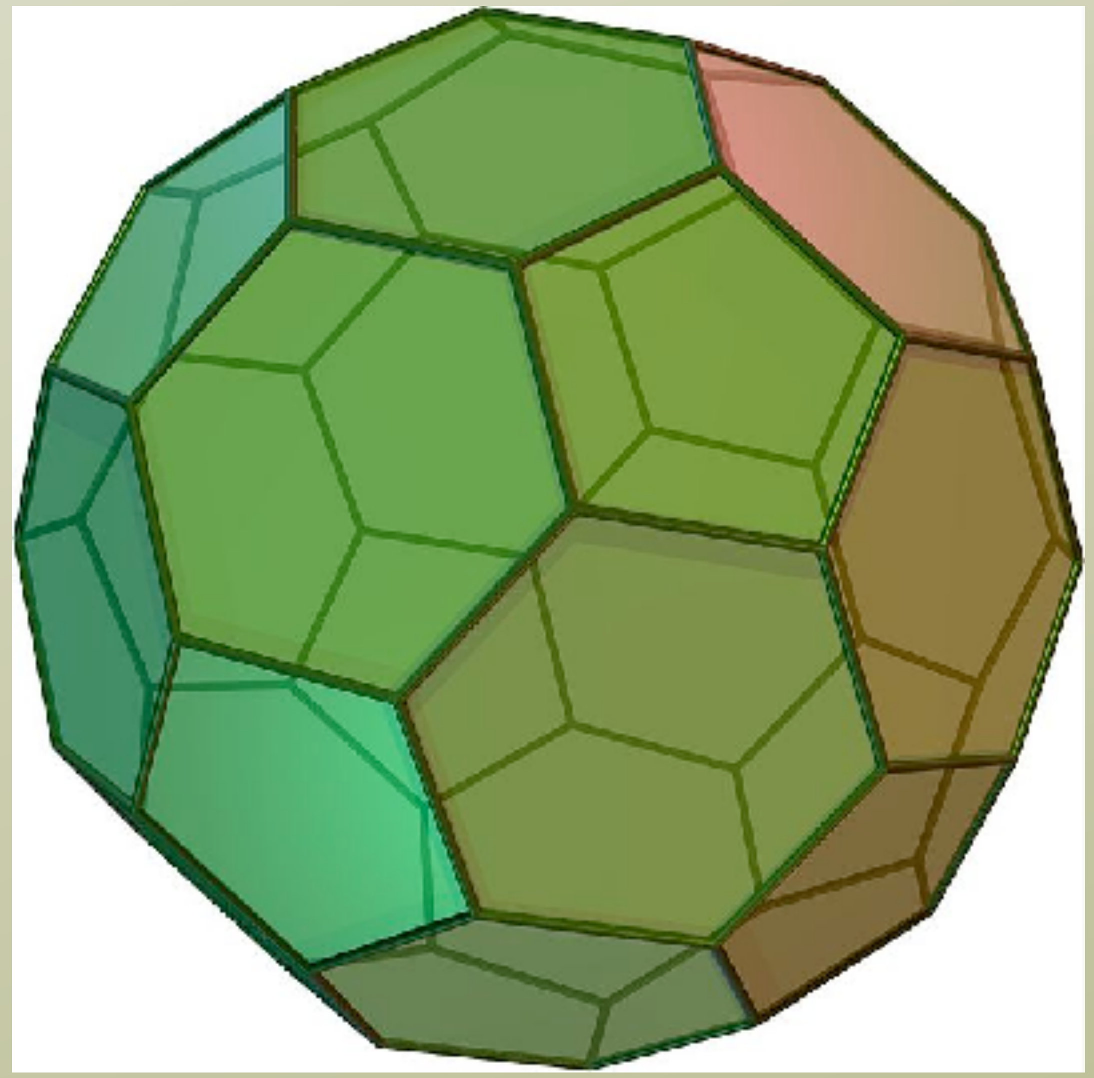
recall…

The dream

The reality



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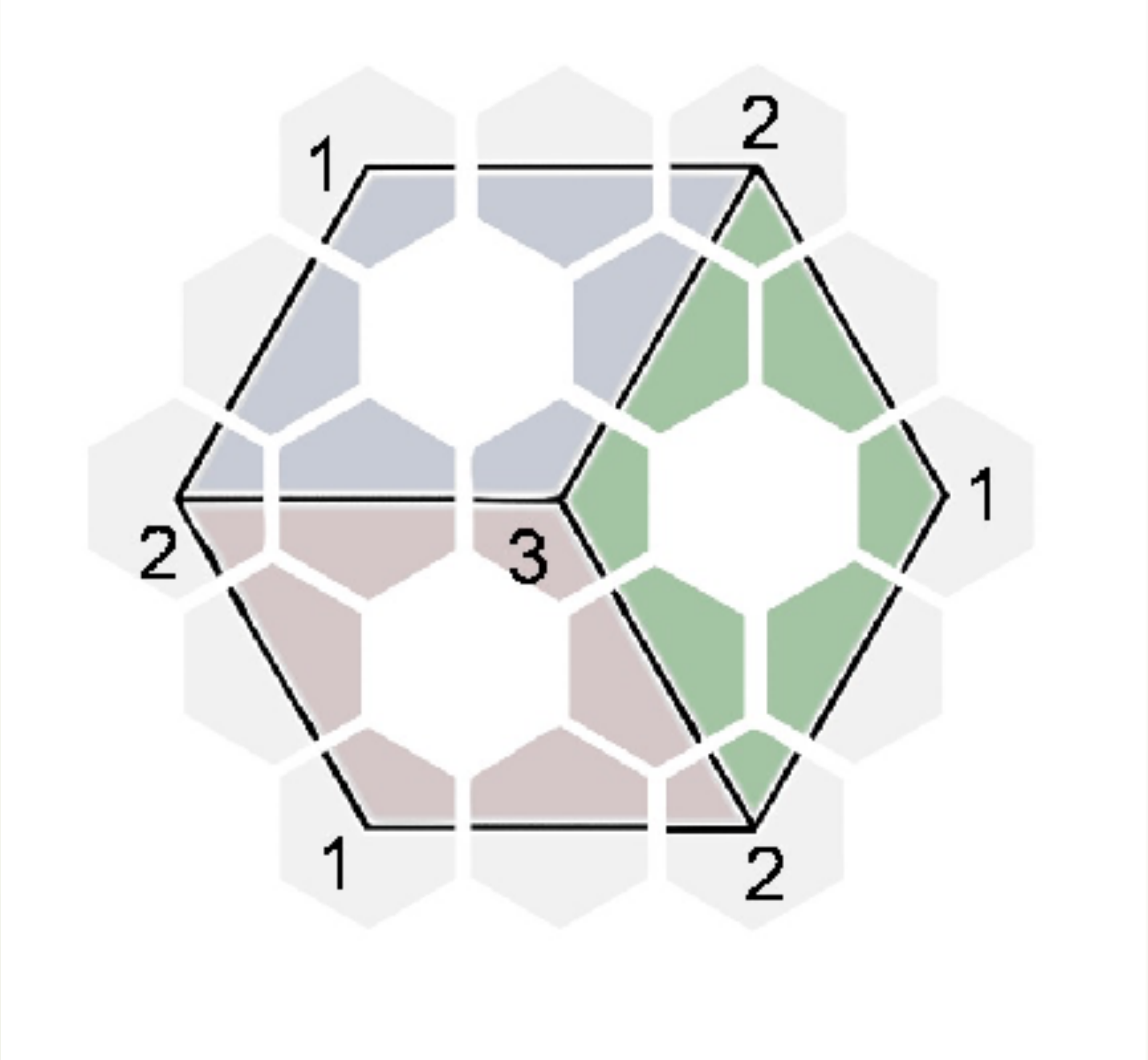
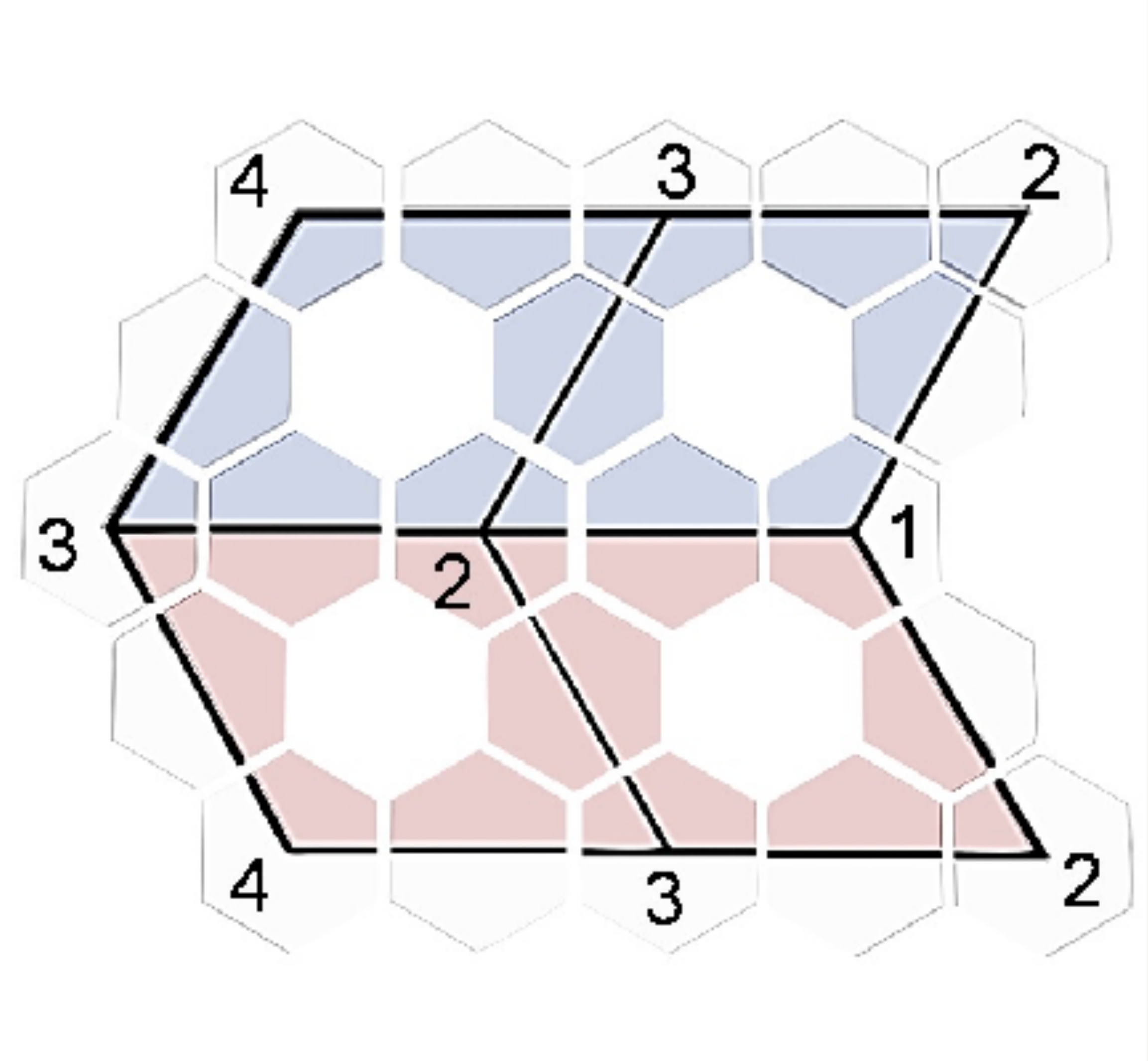
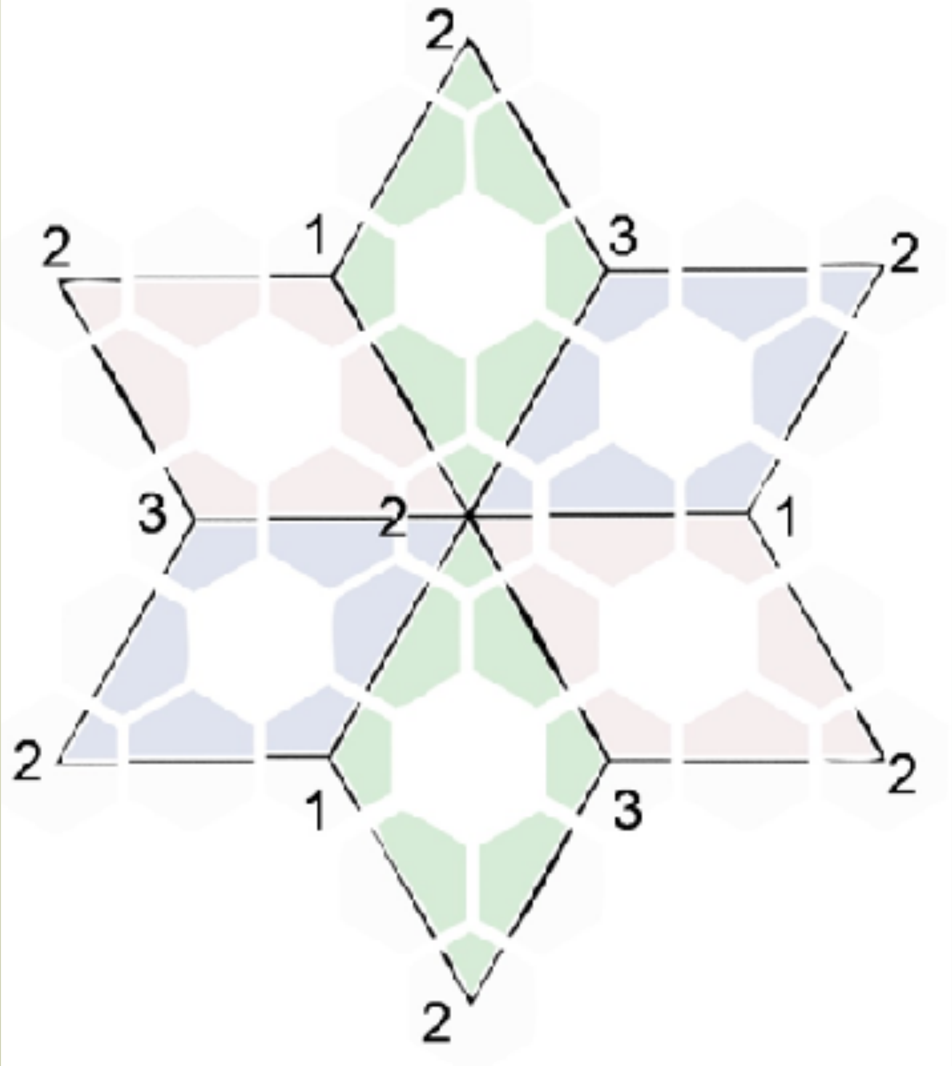
(flat) plane diamond packing vertex arrangements







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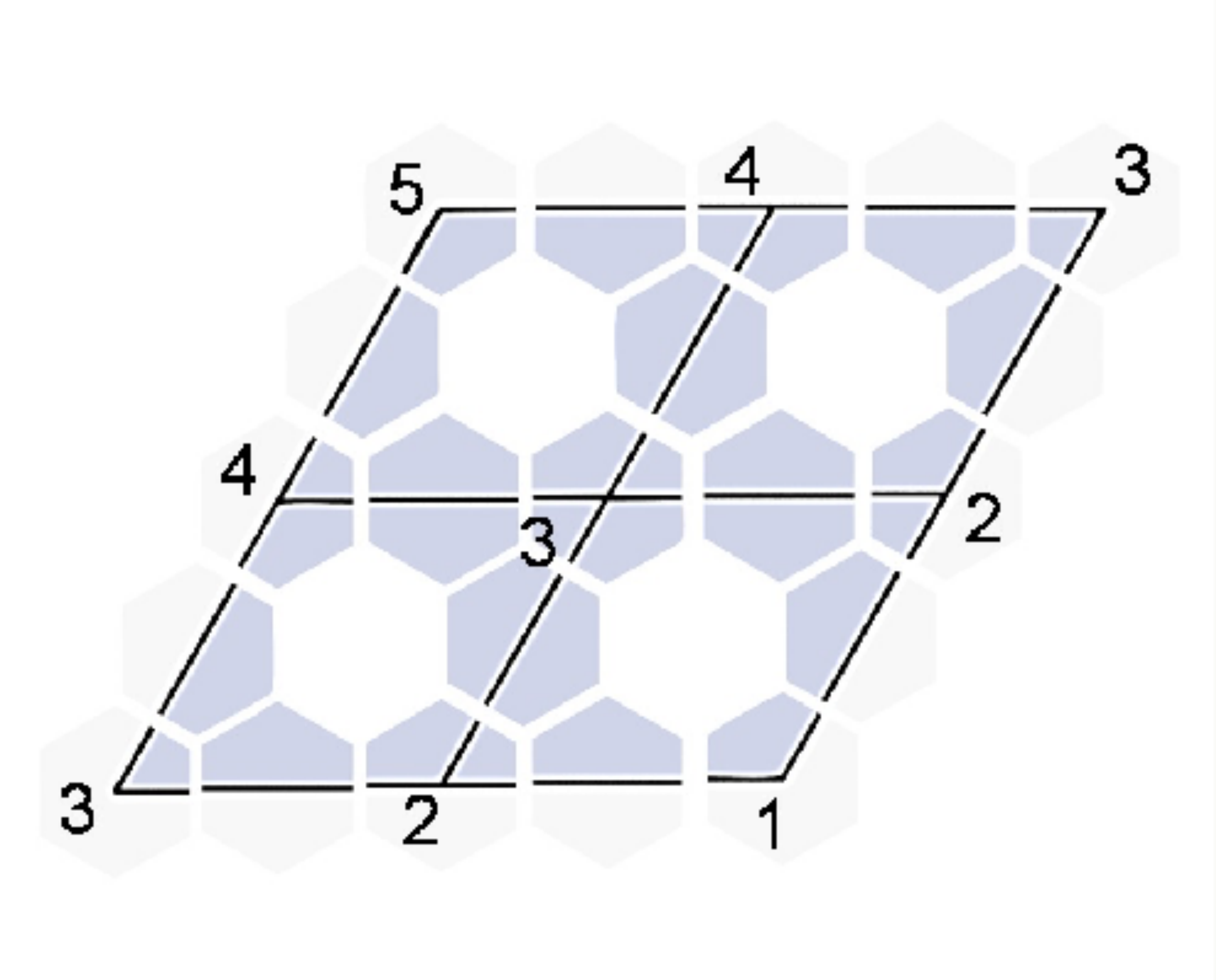

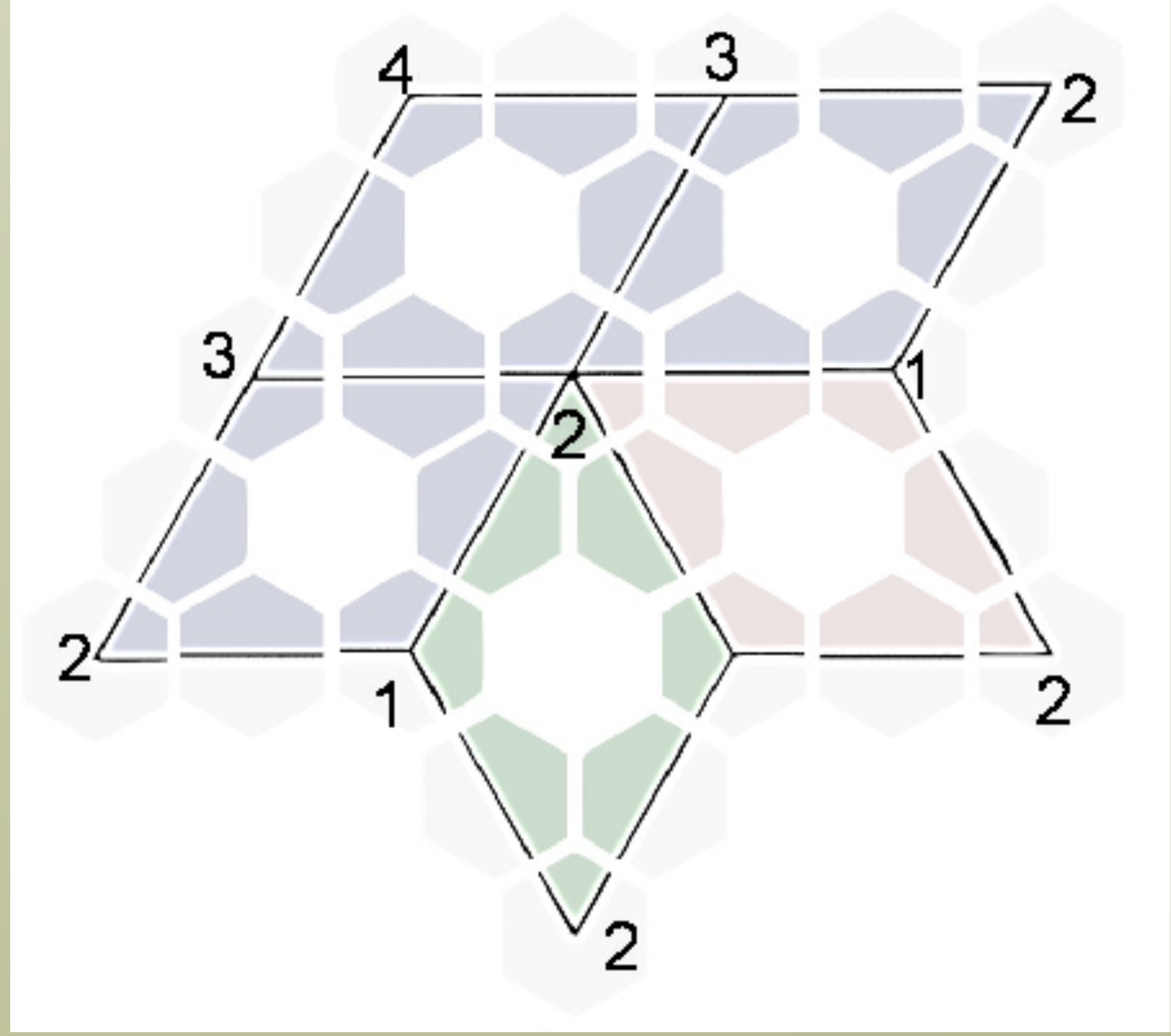

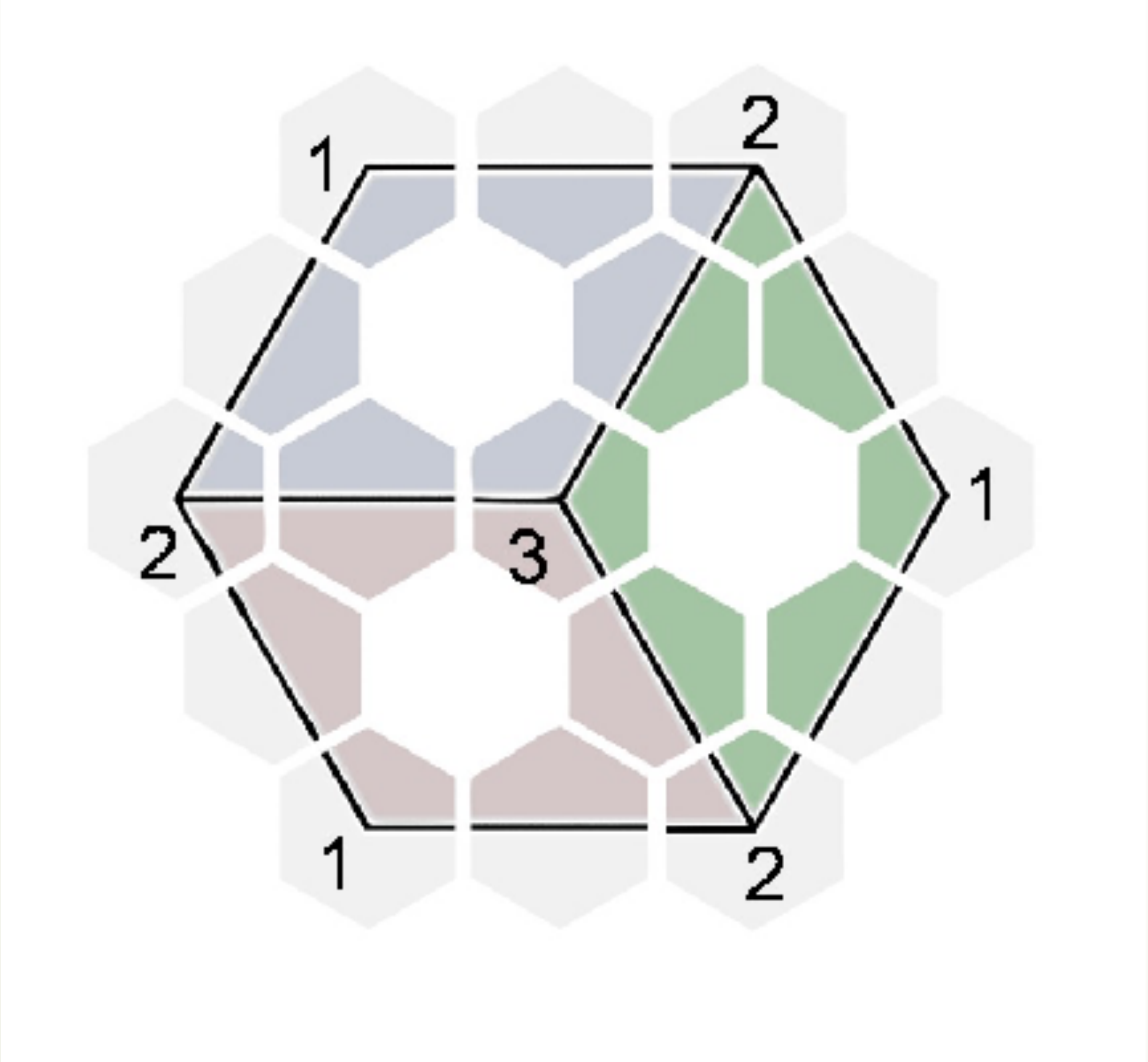
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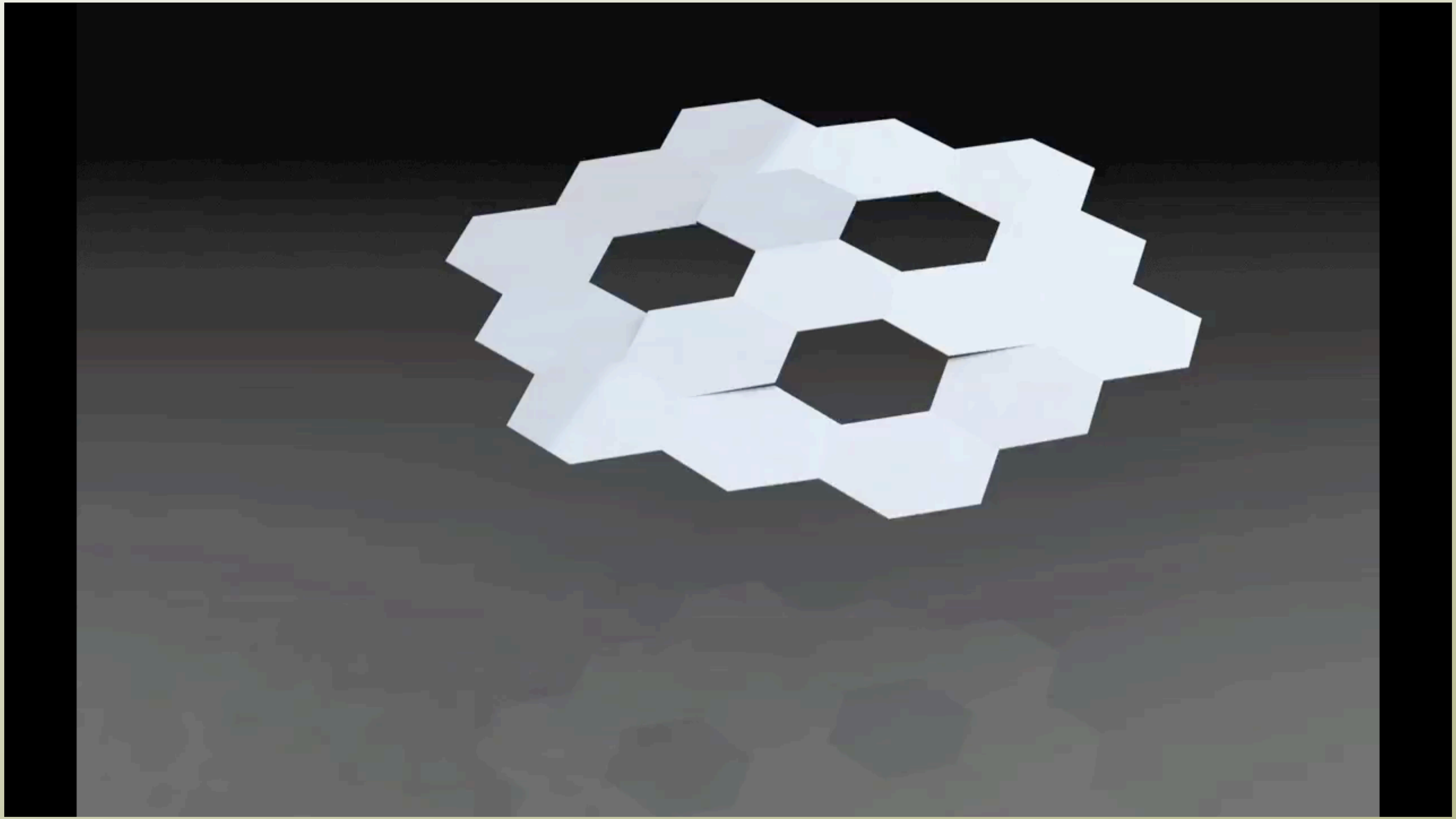

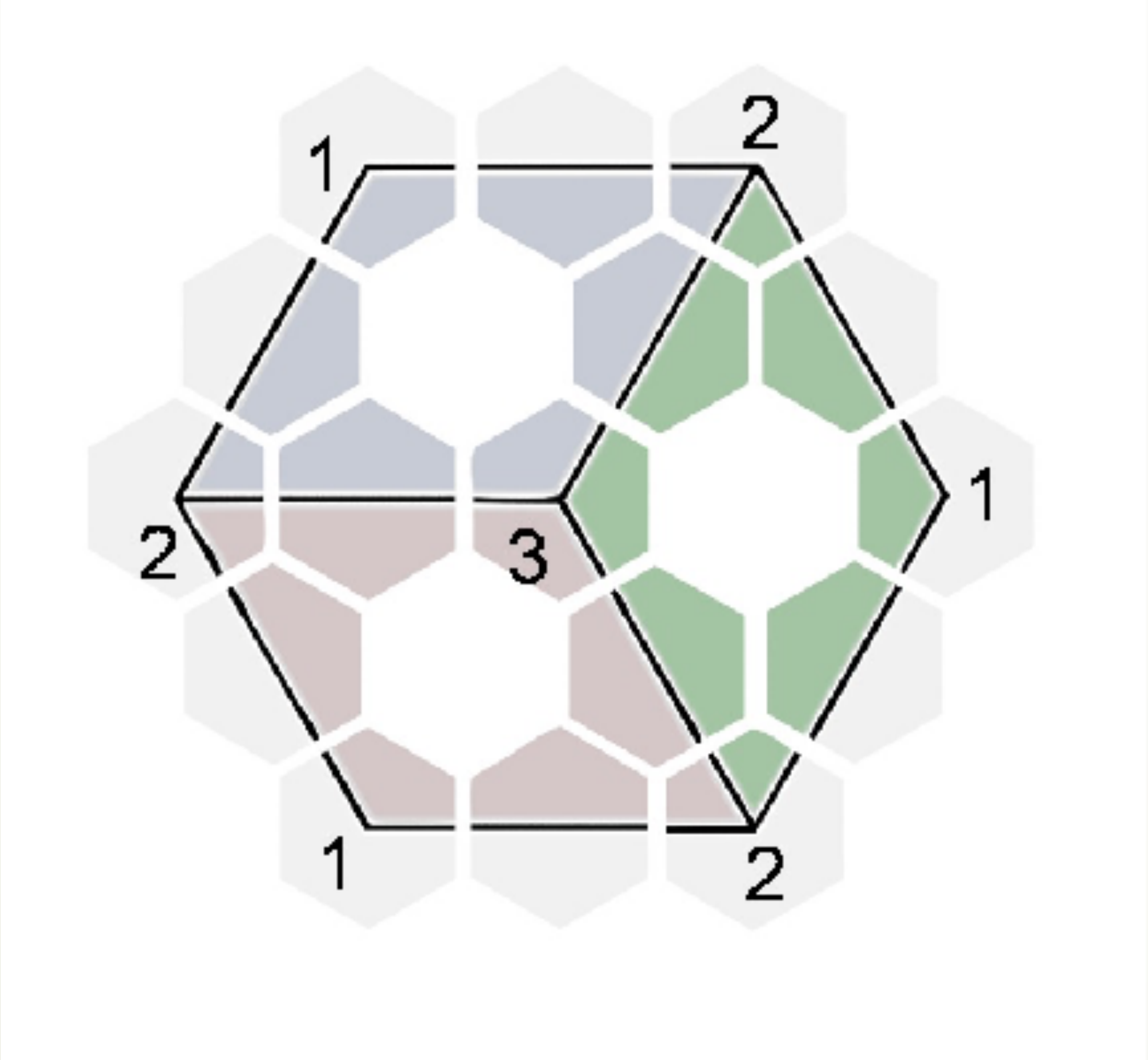
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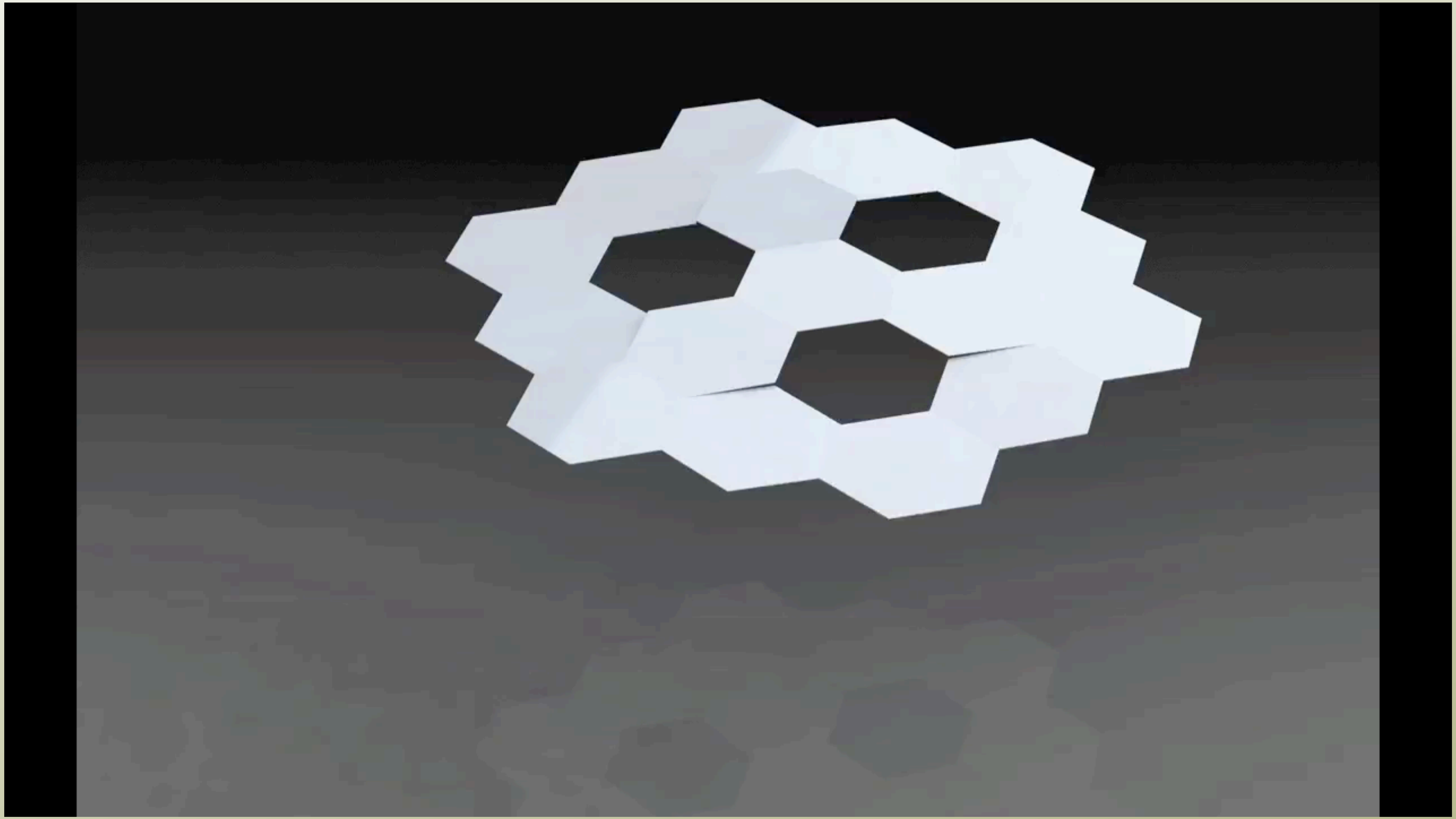

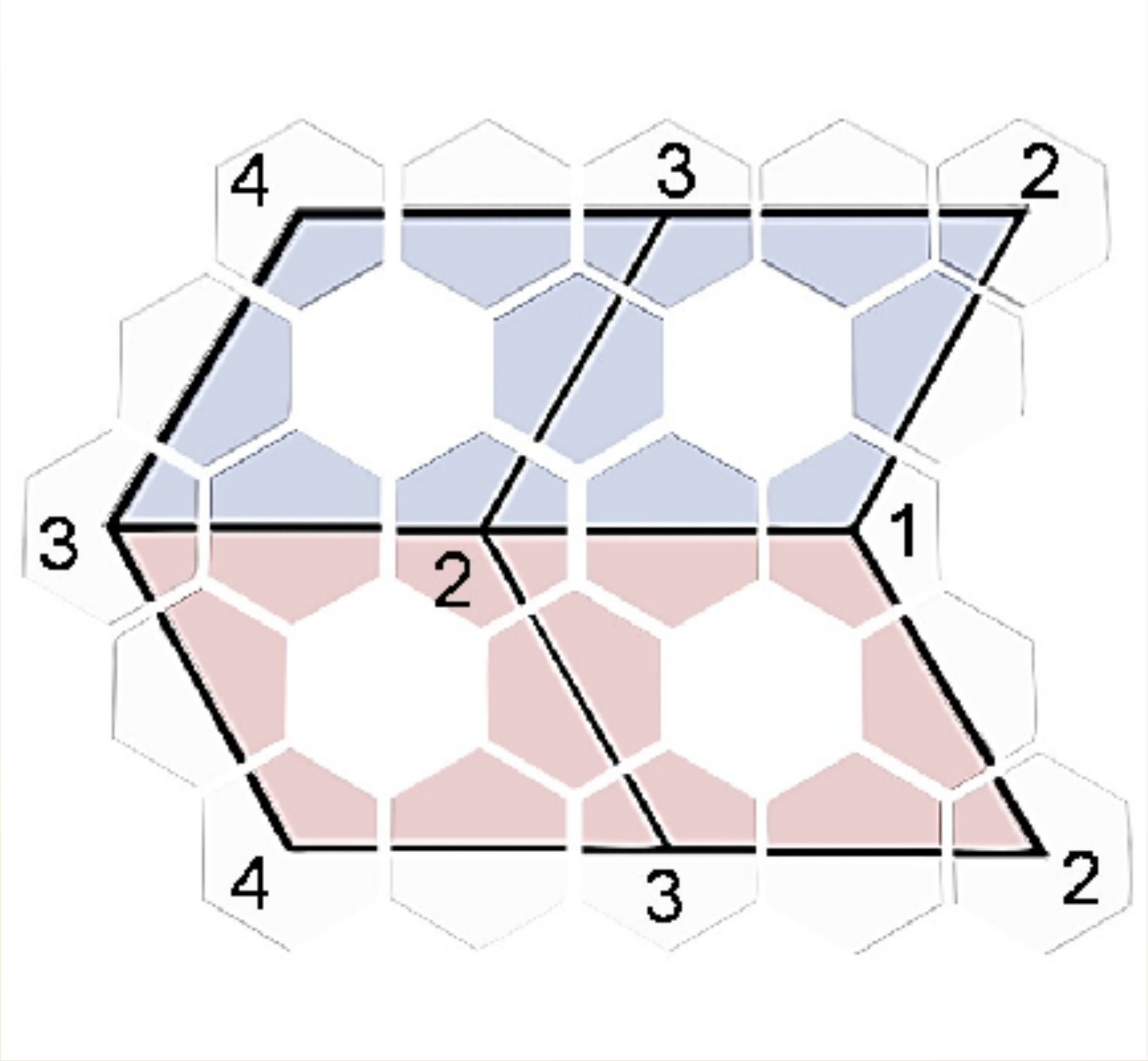

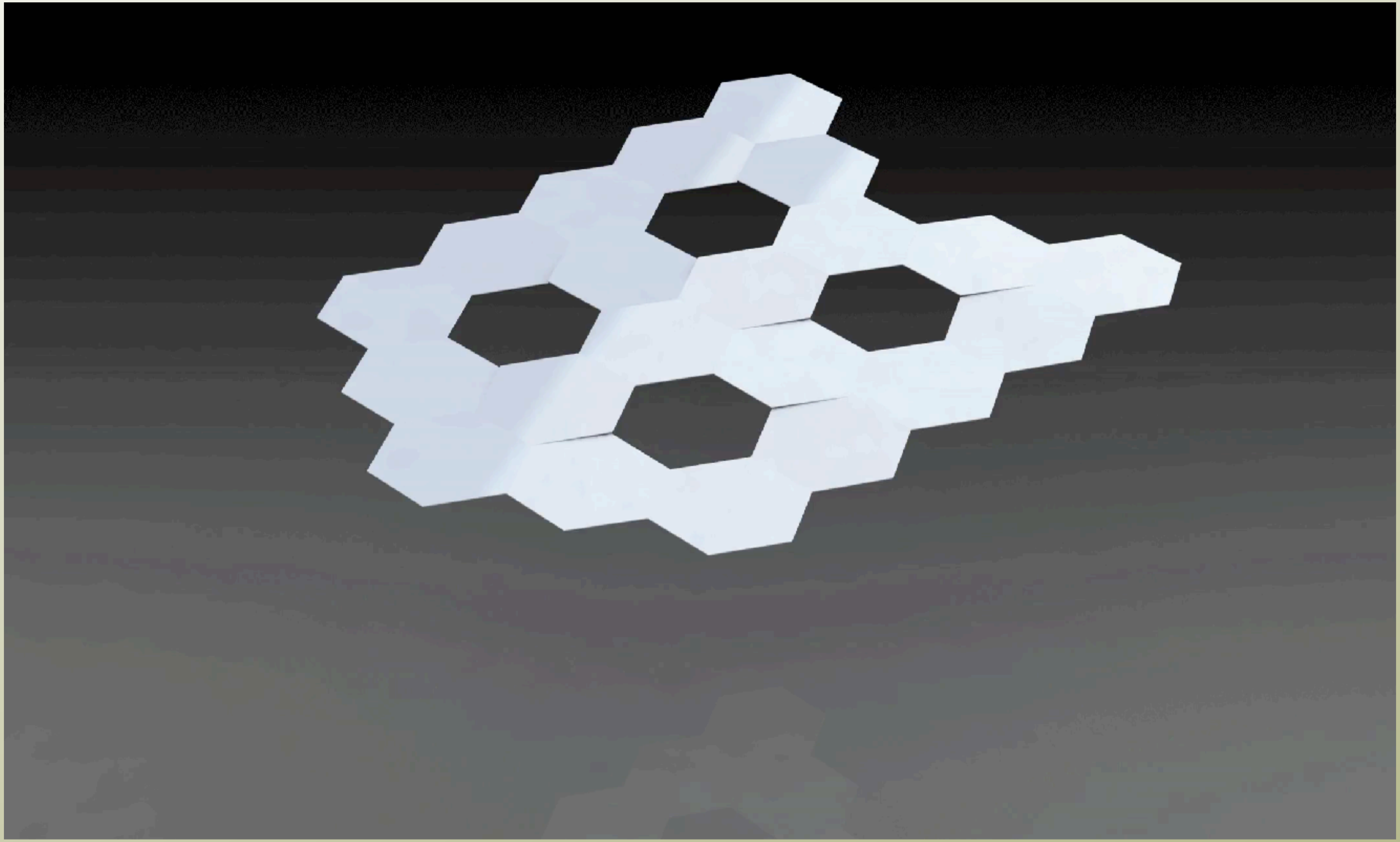

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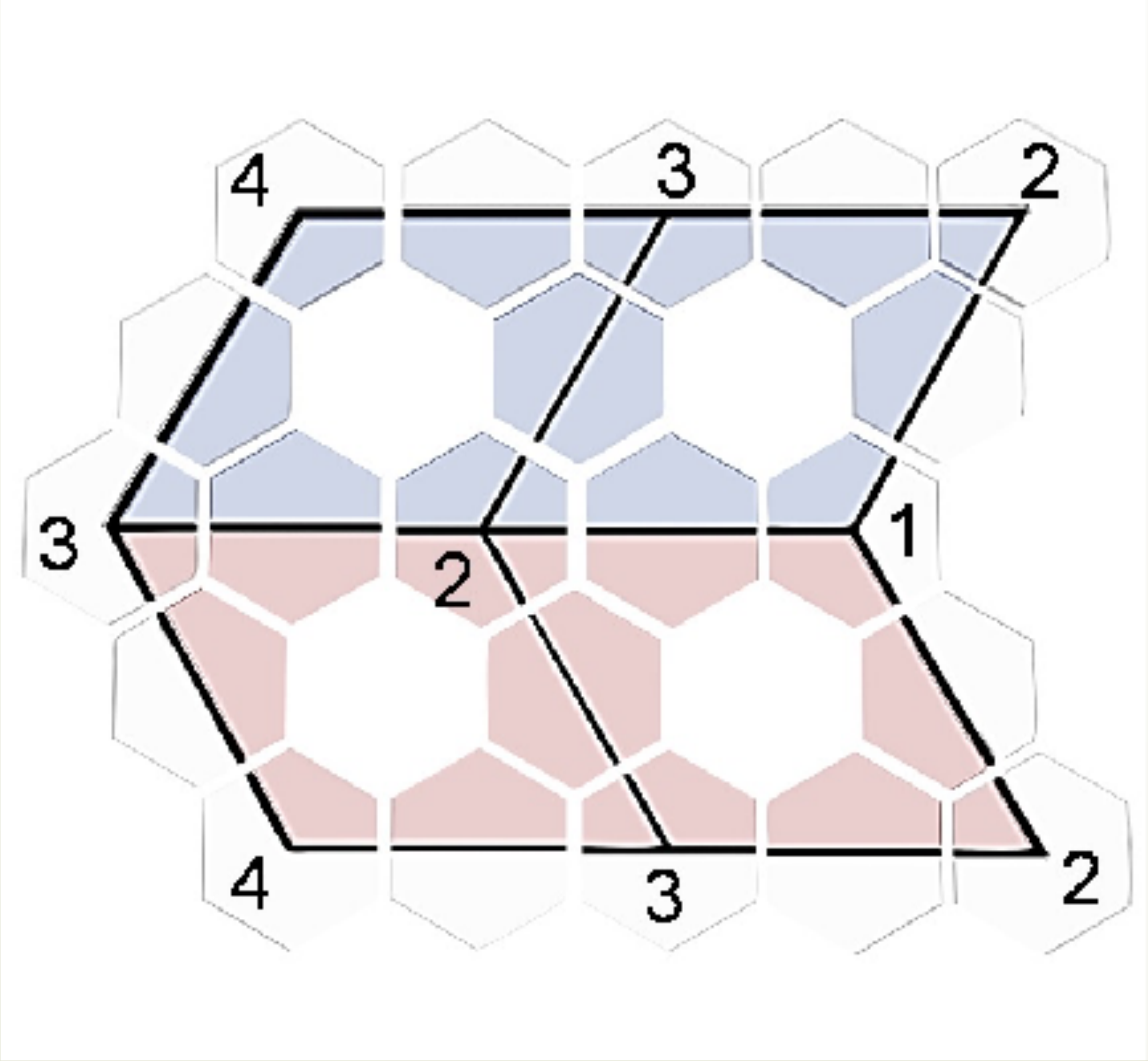
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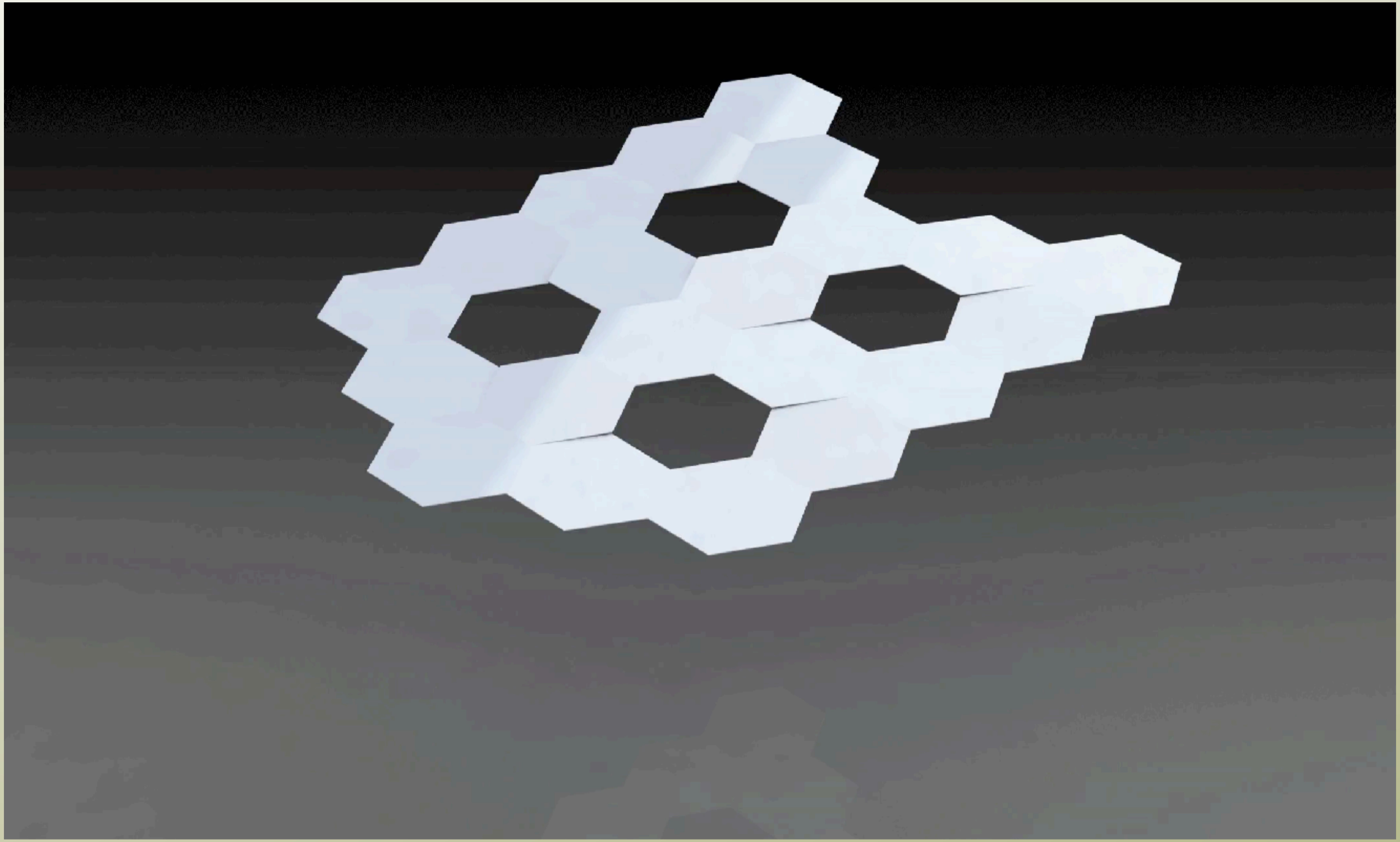

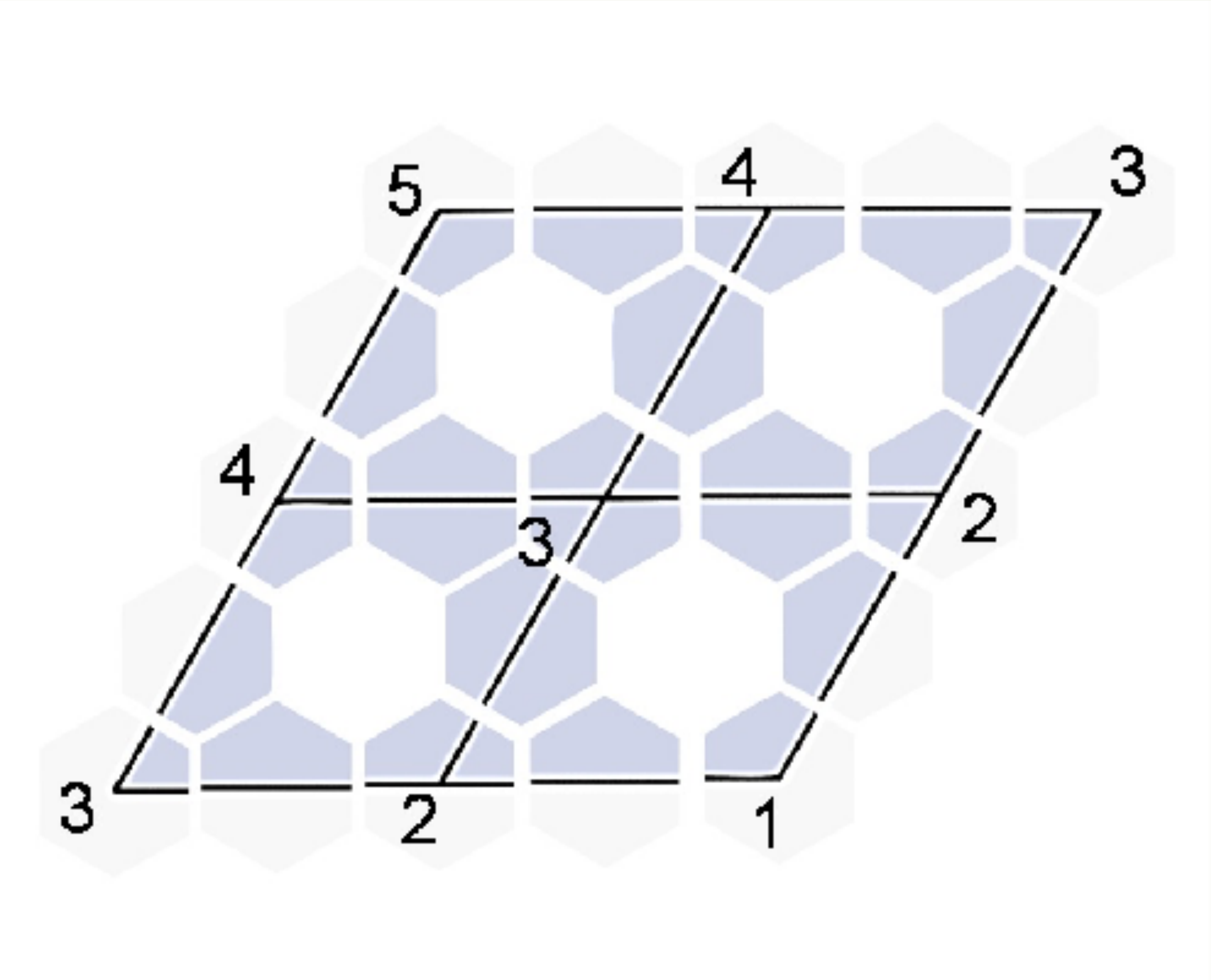

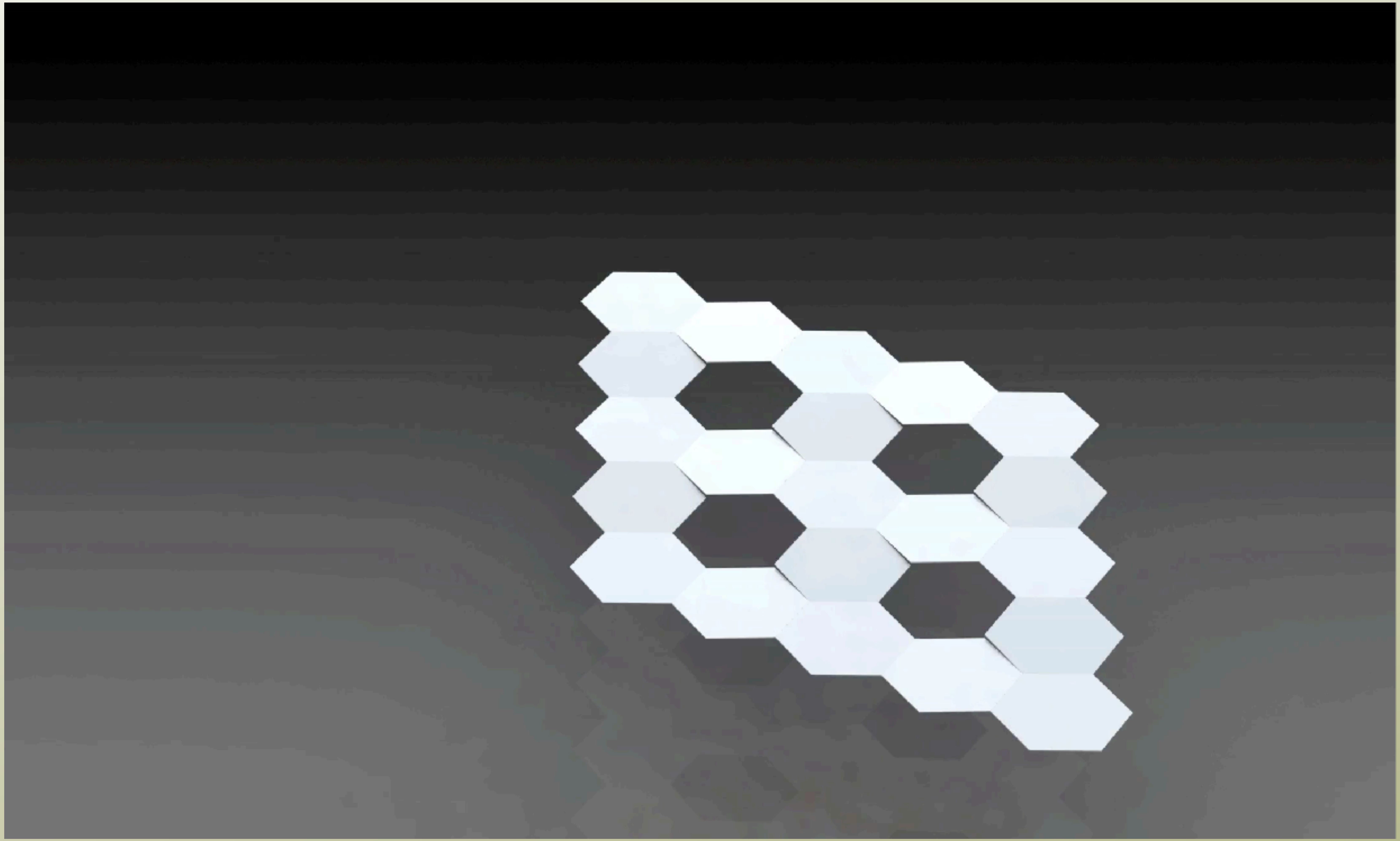

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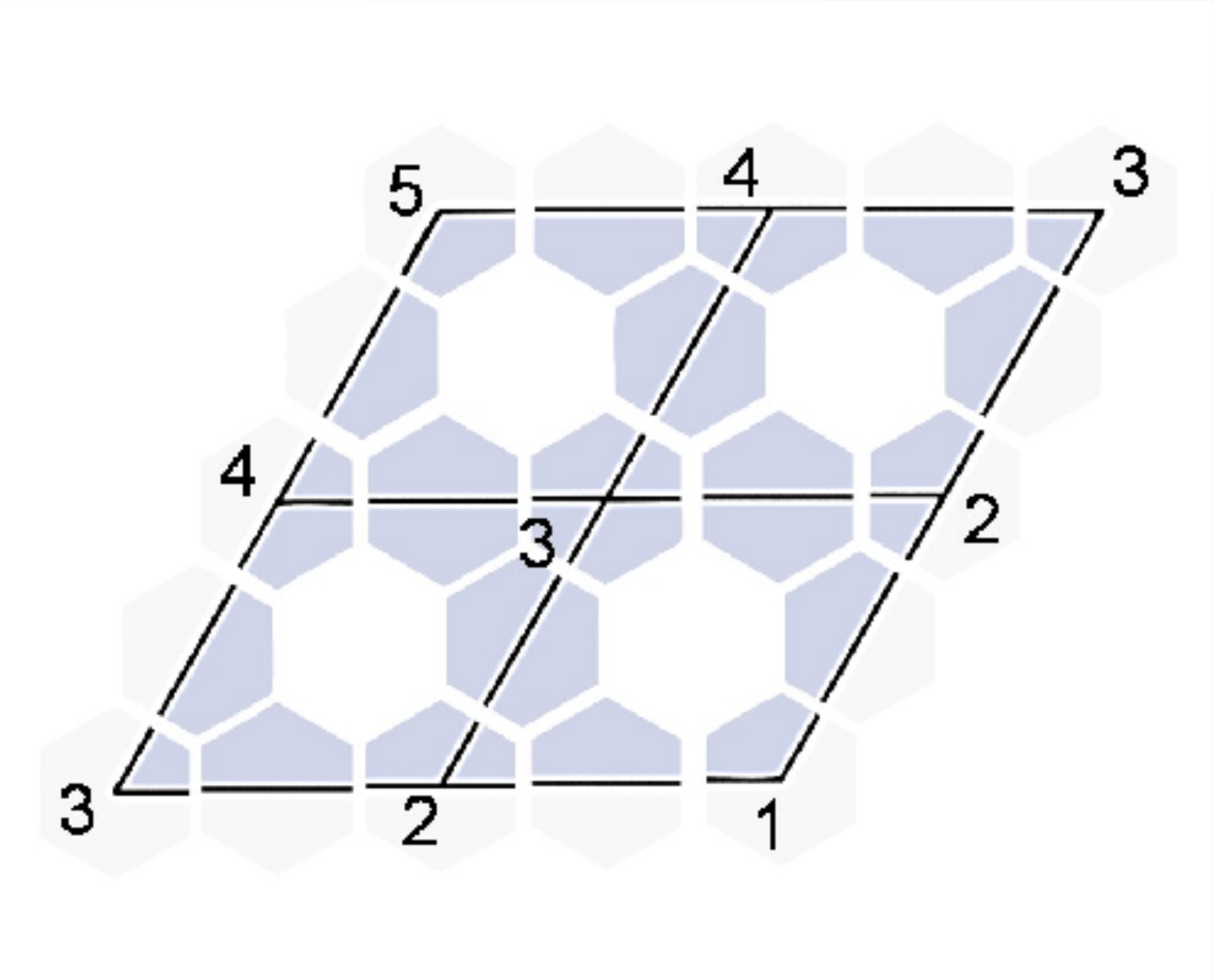
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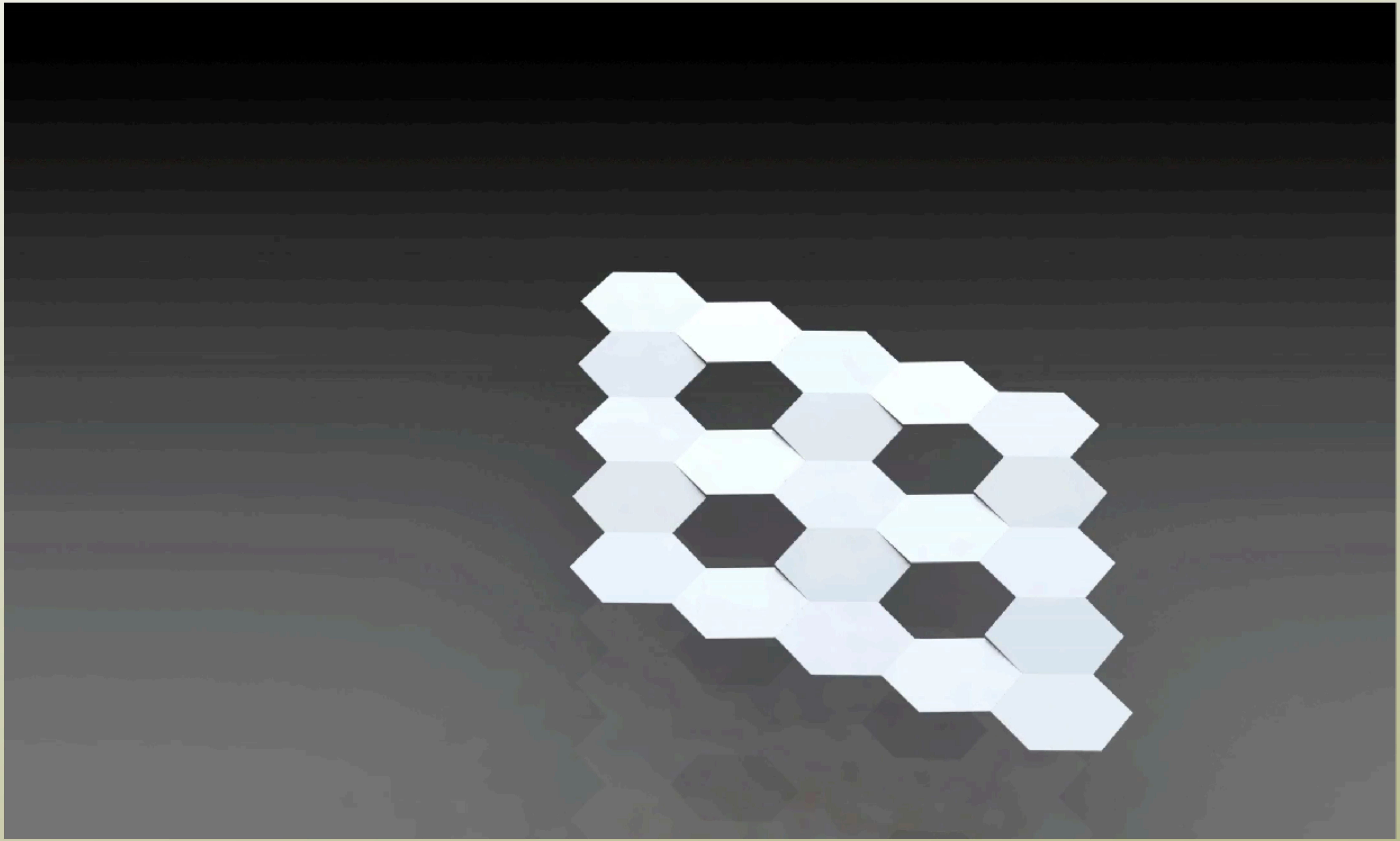

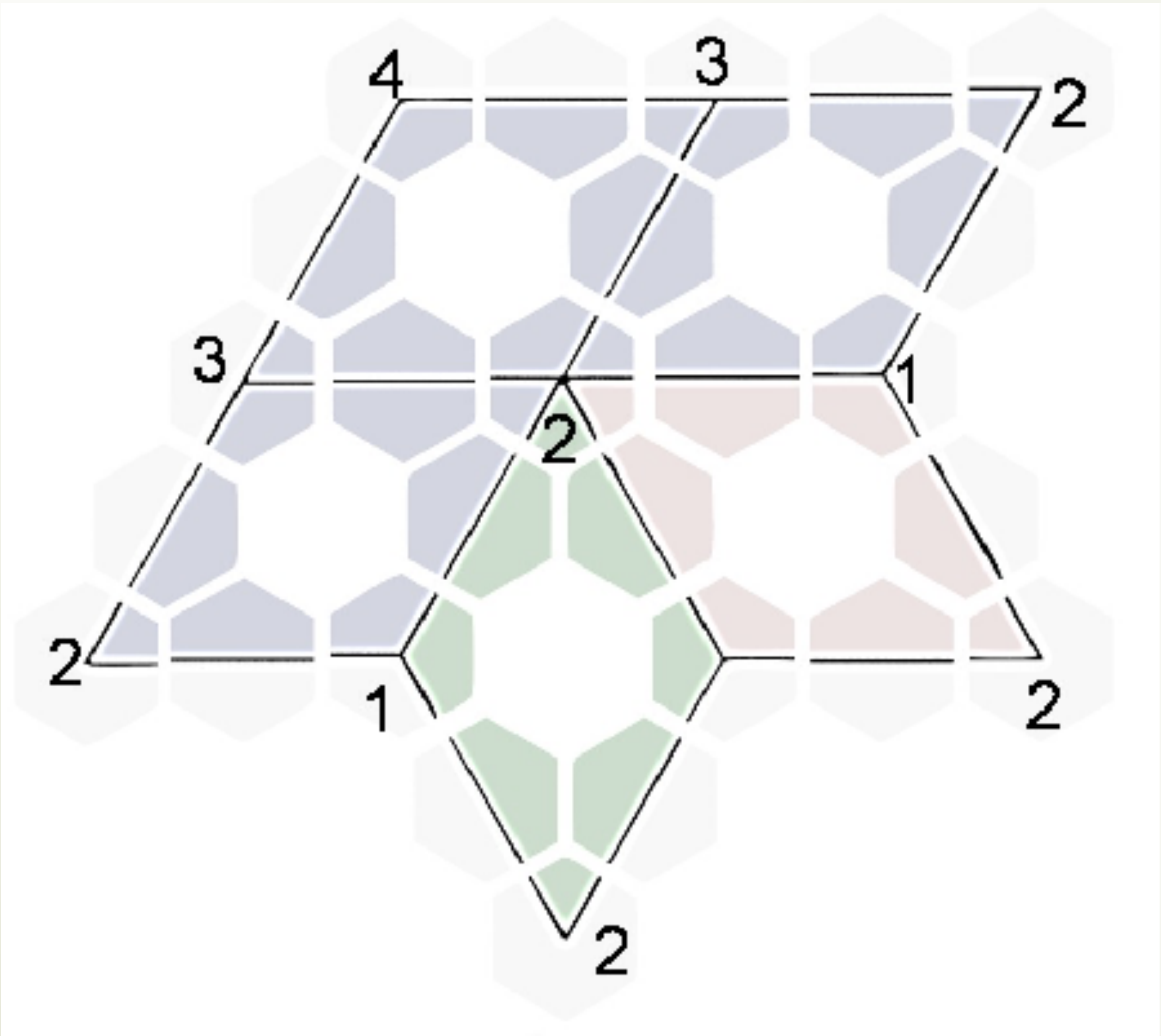
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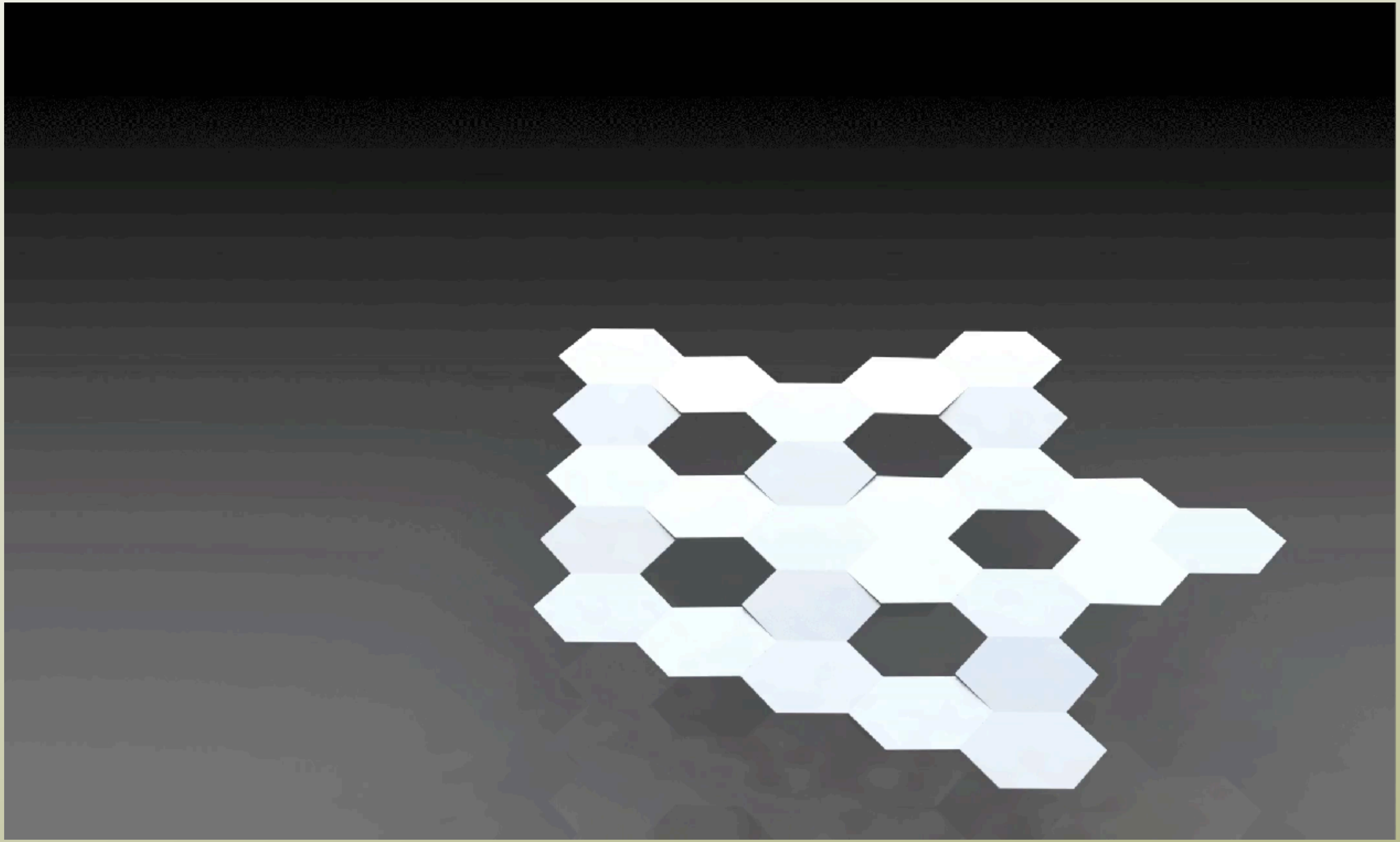

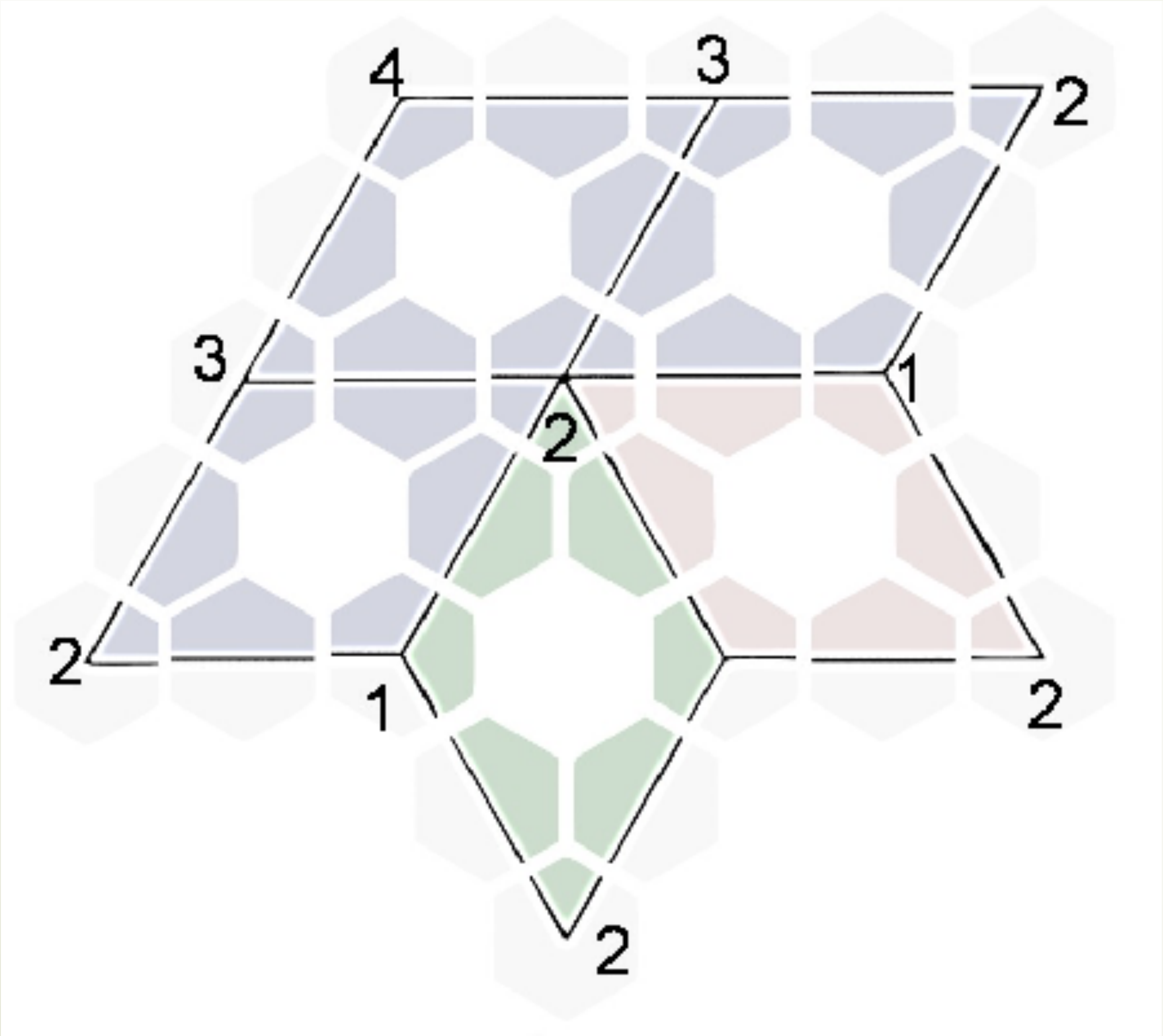
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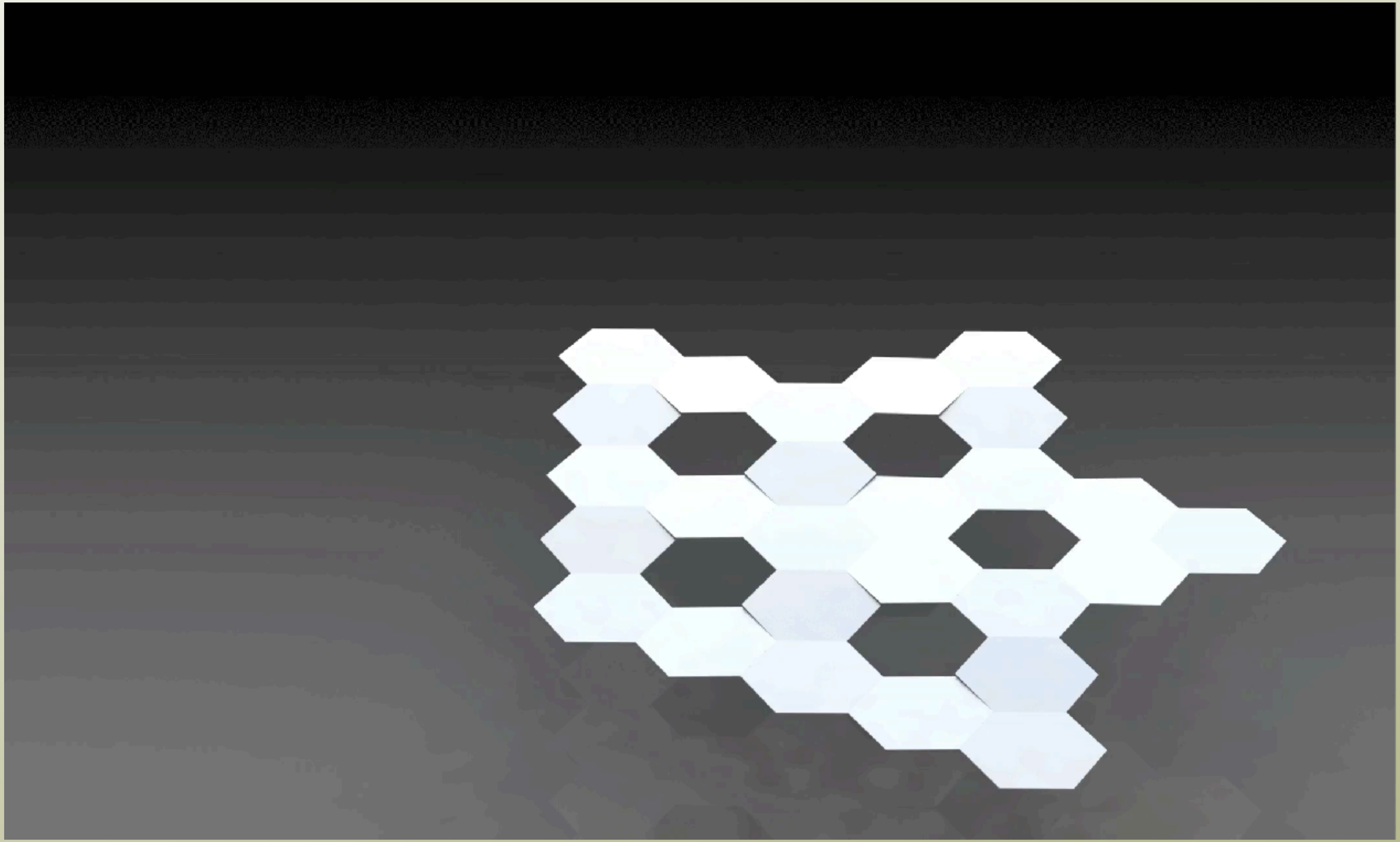

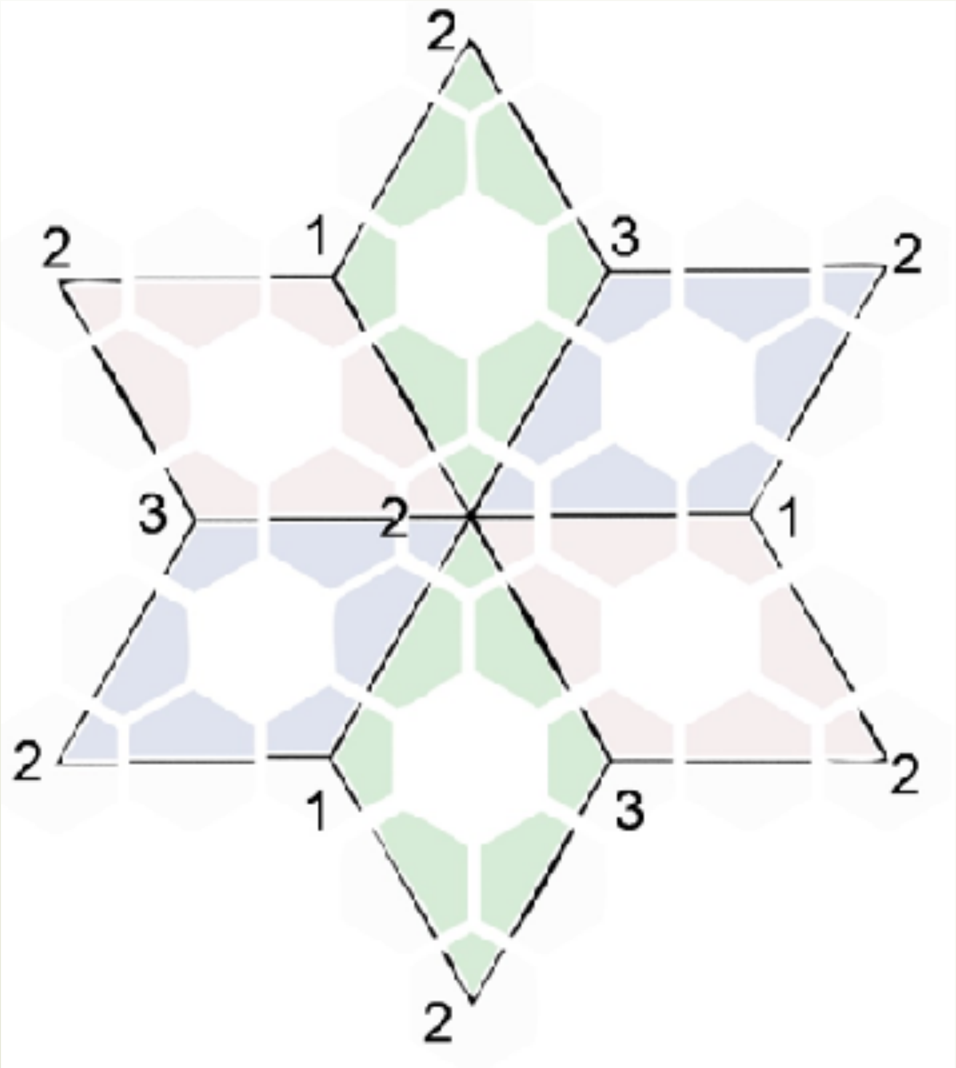
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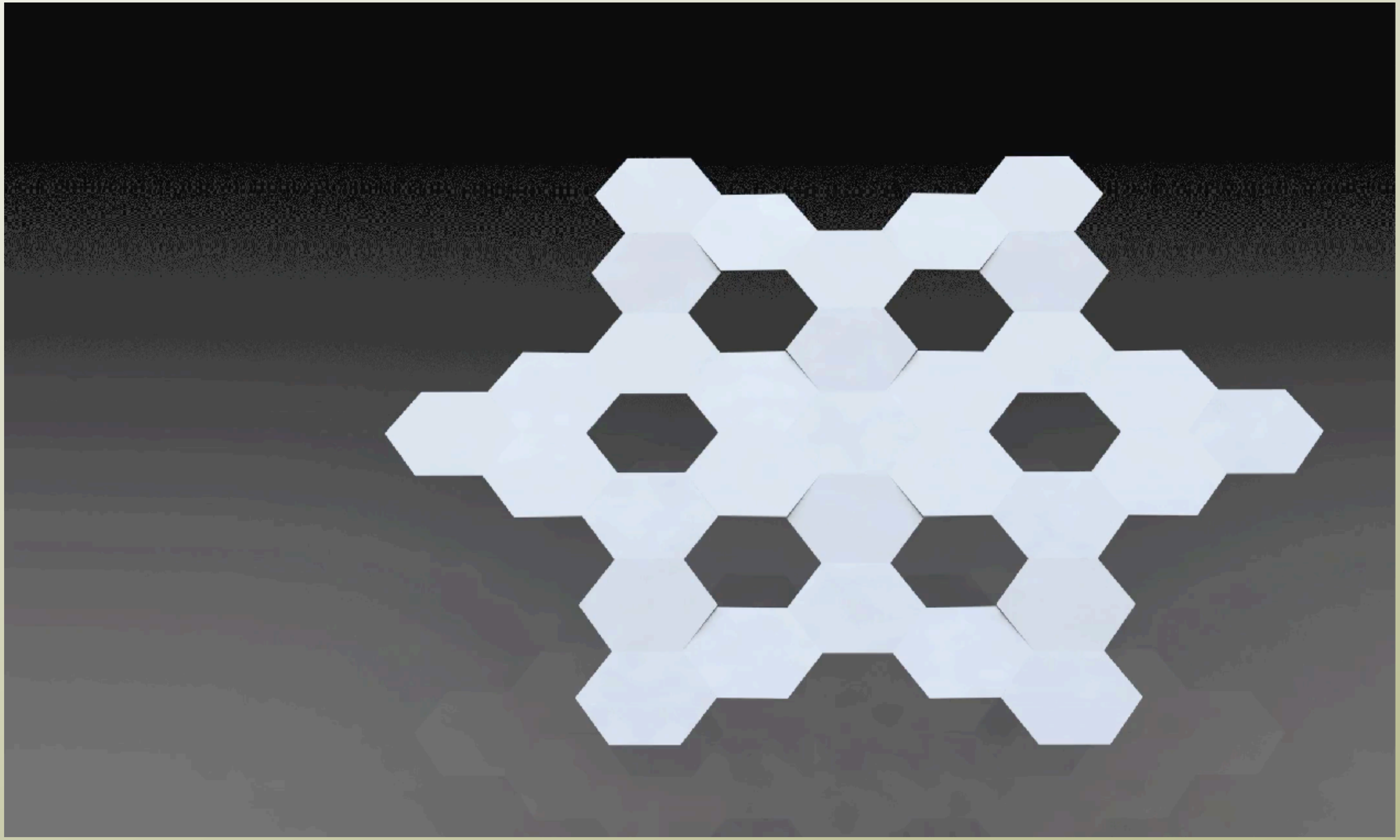

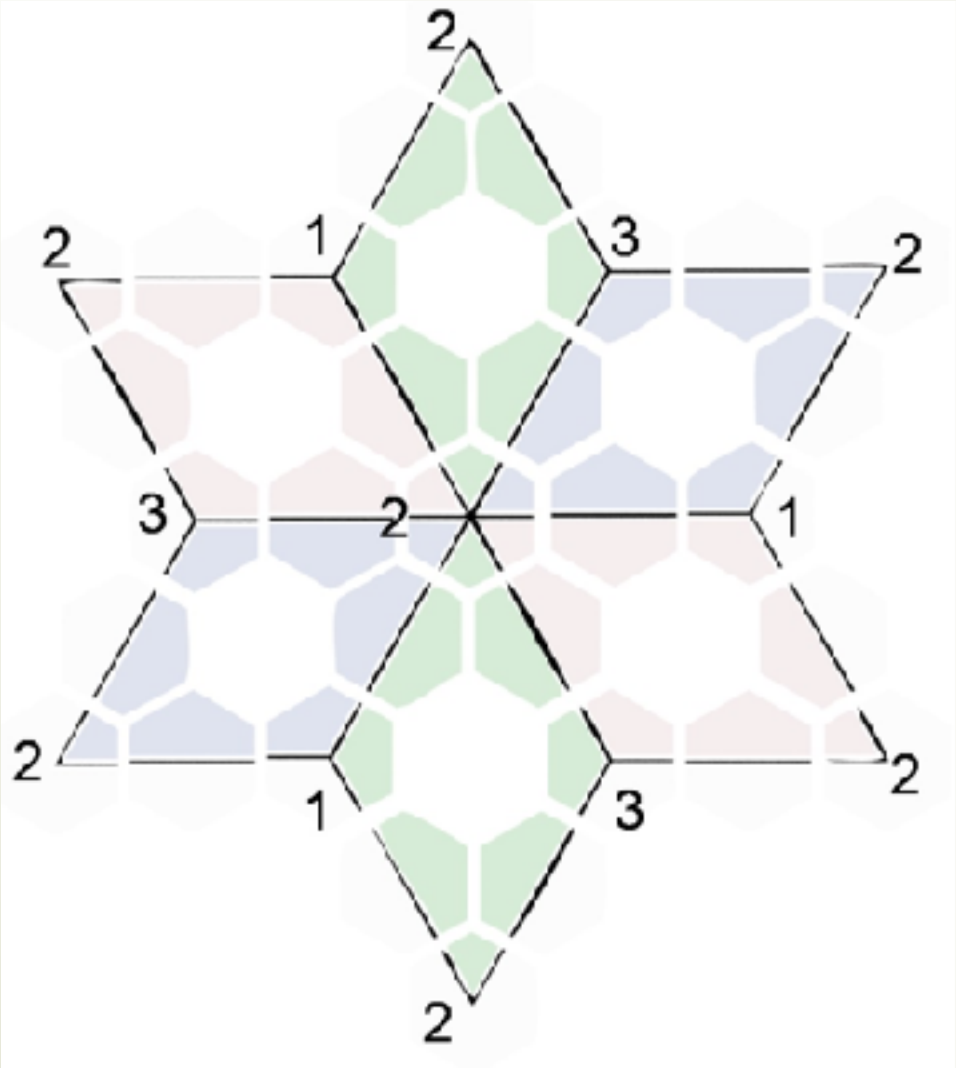
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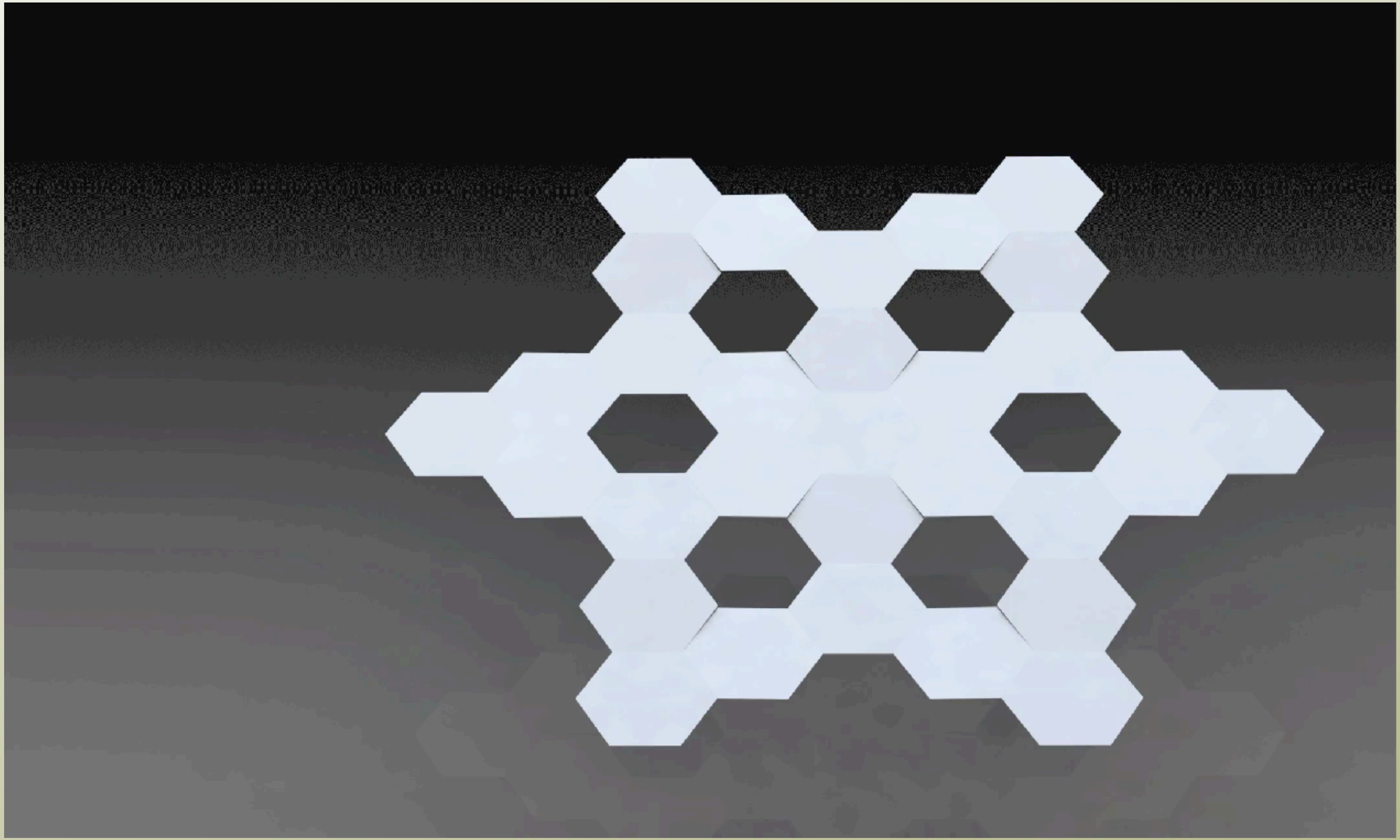
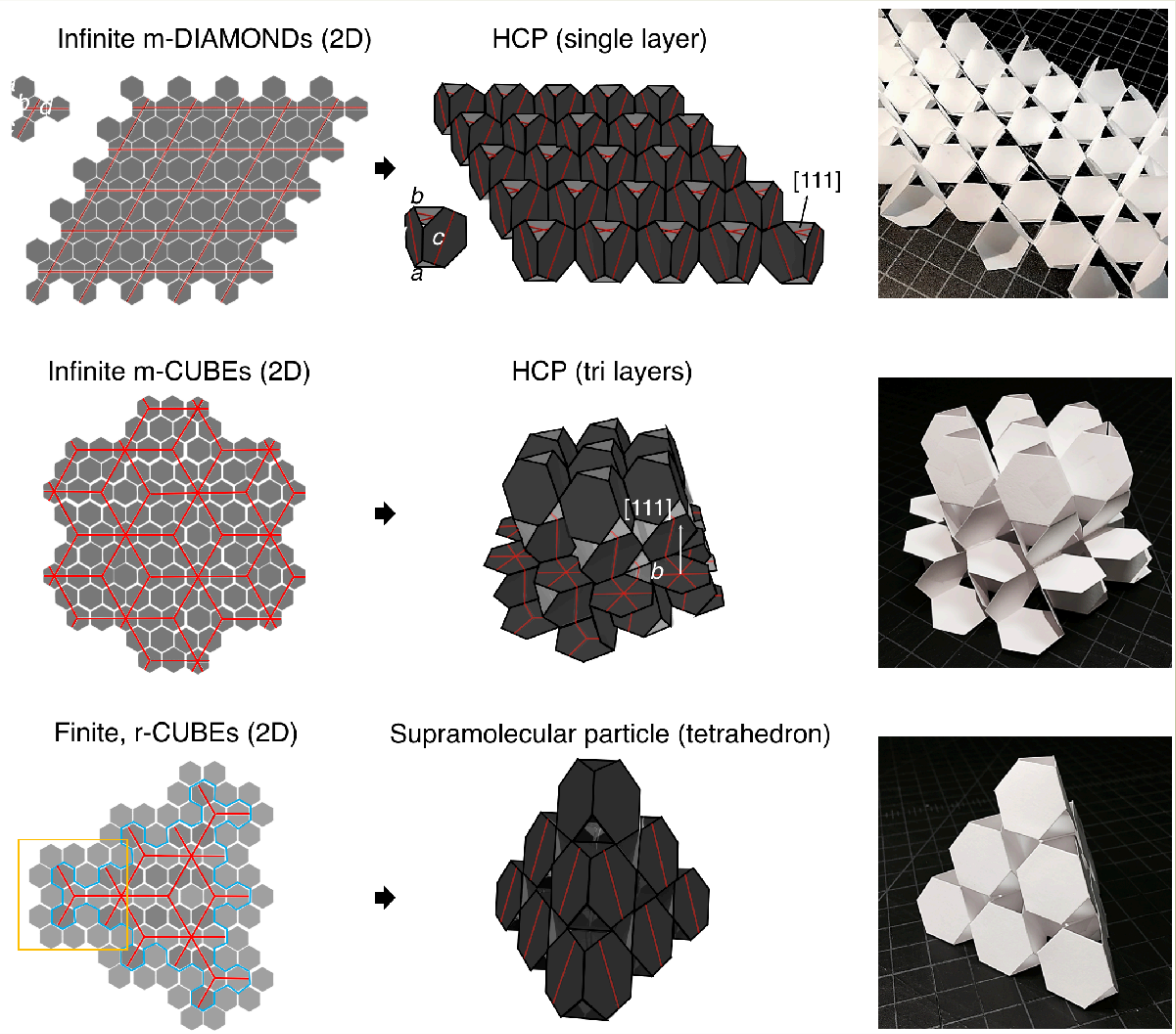
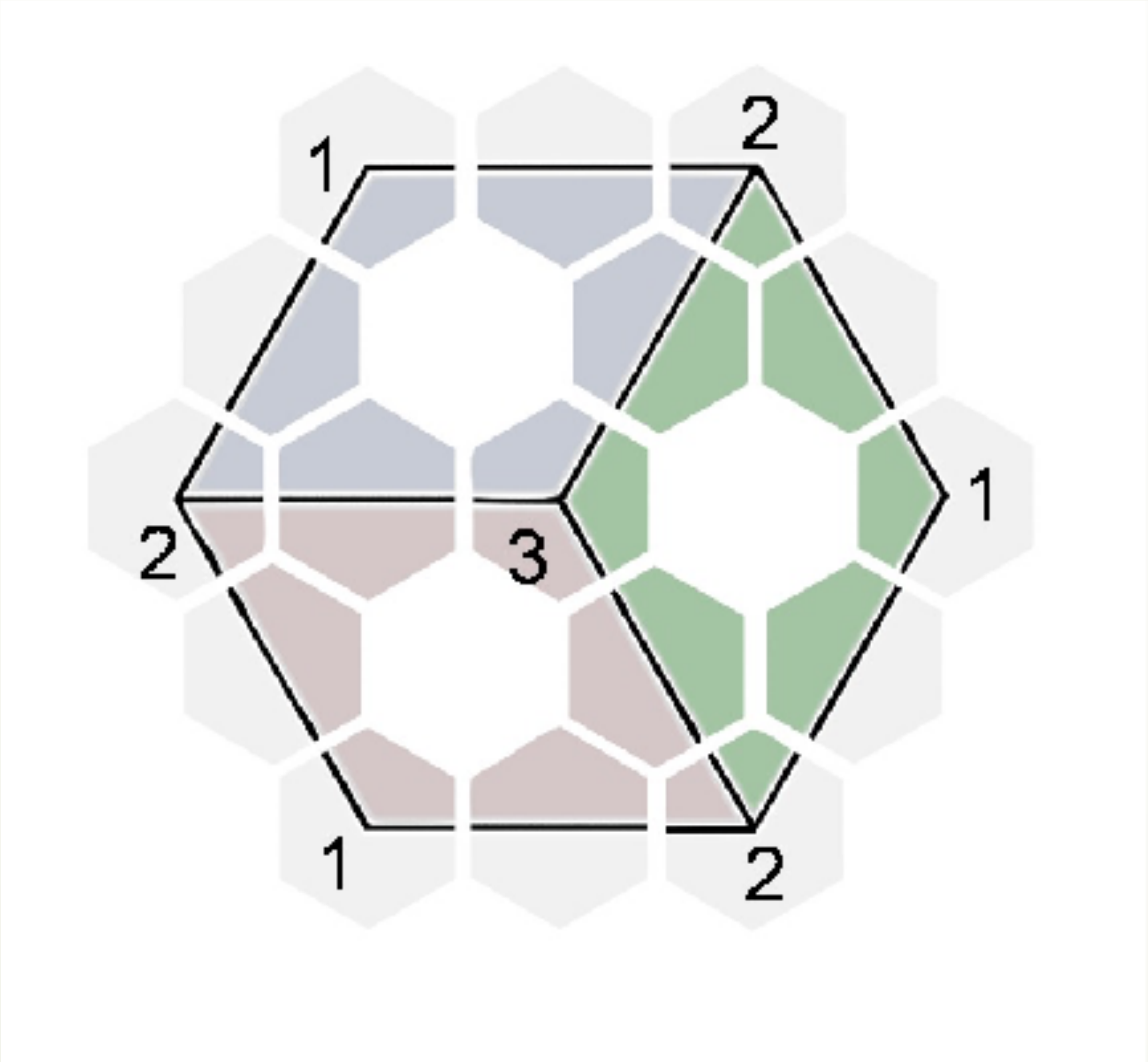
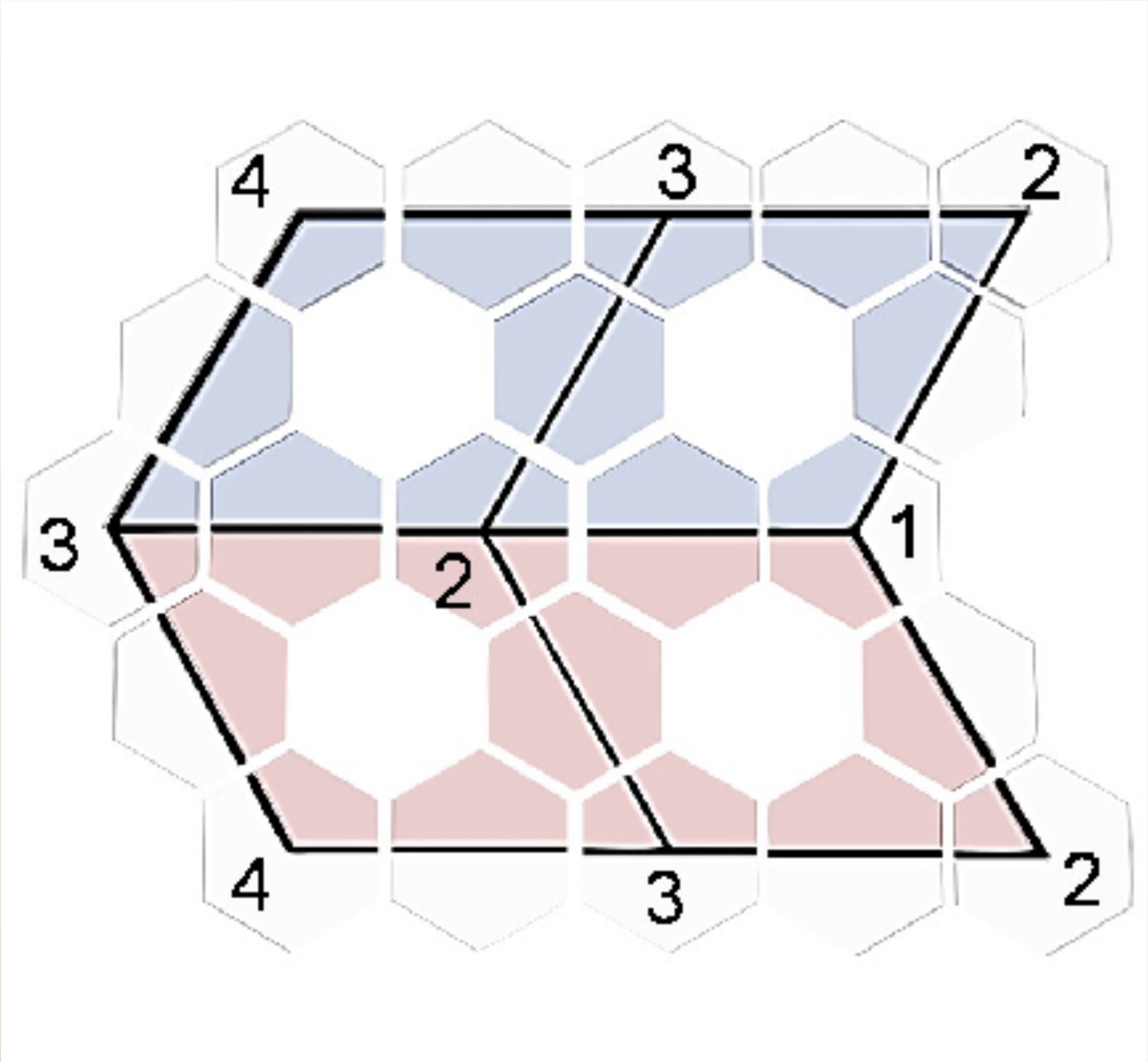
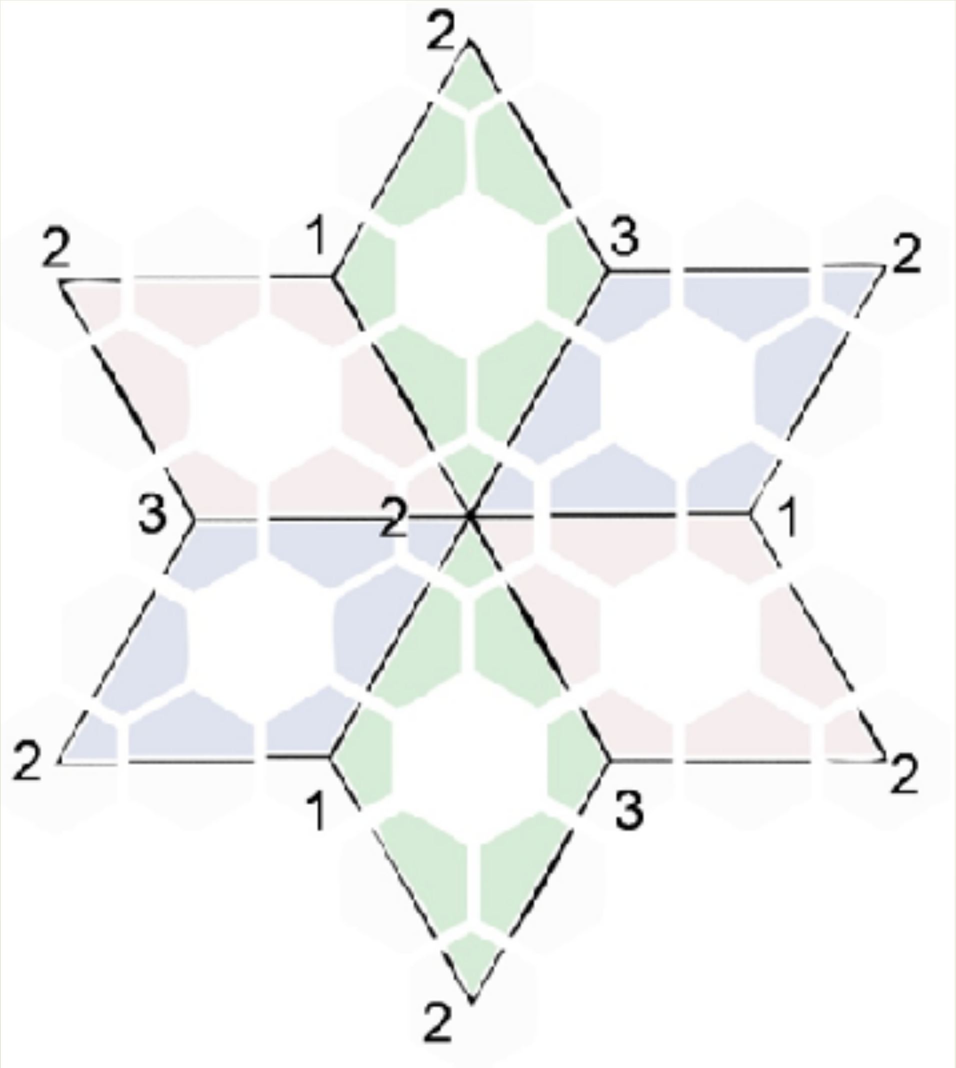


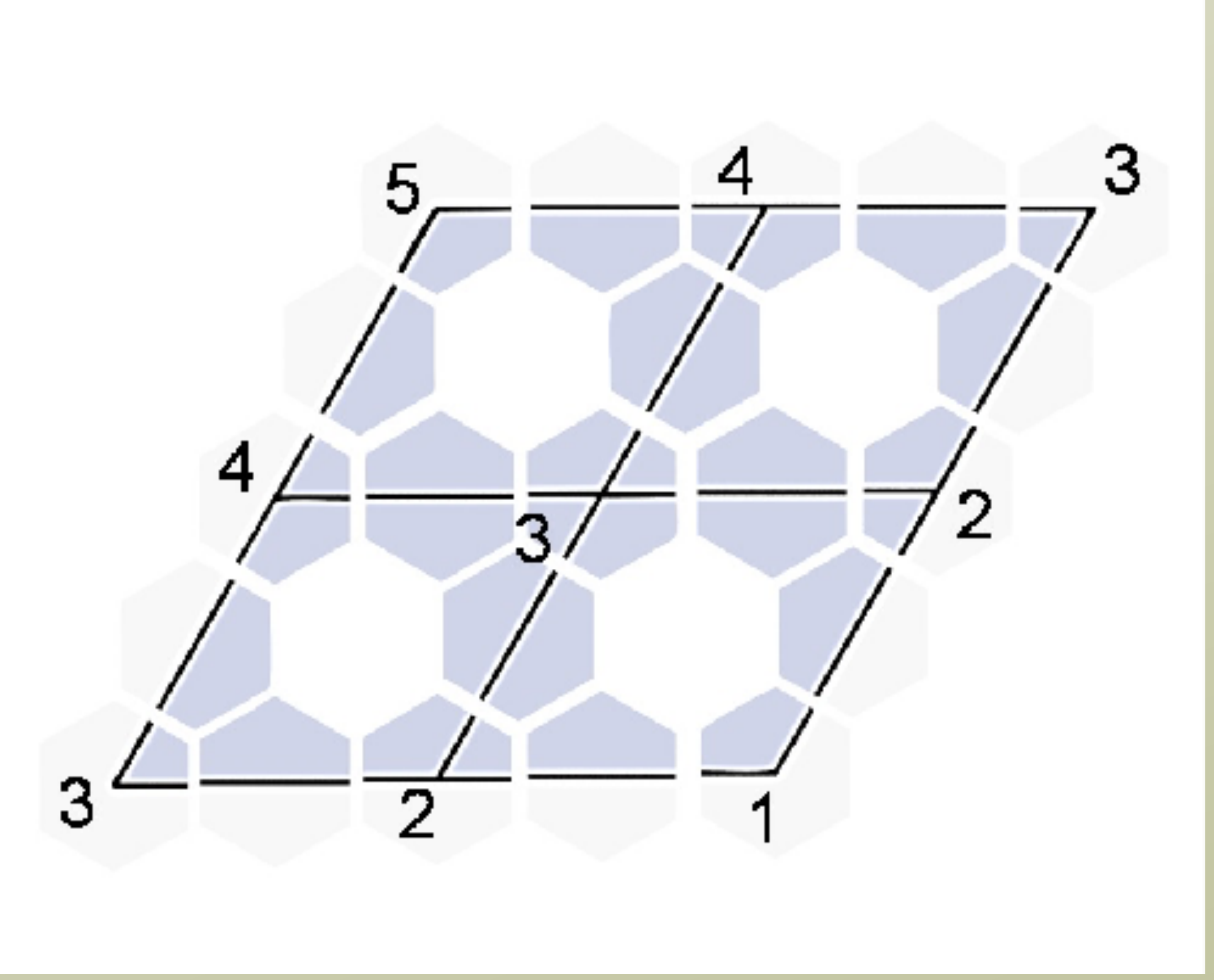


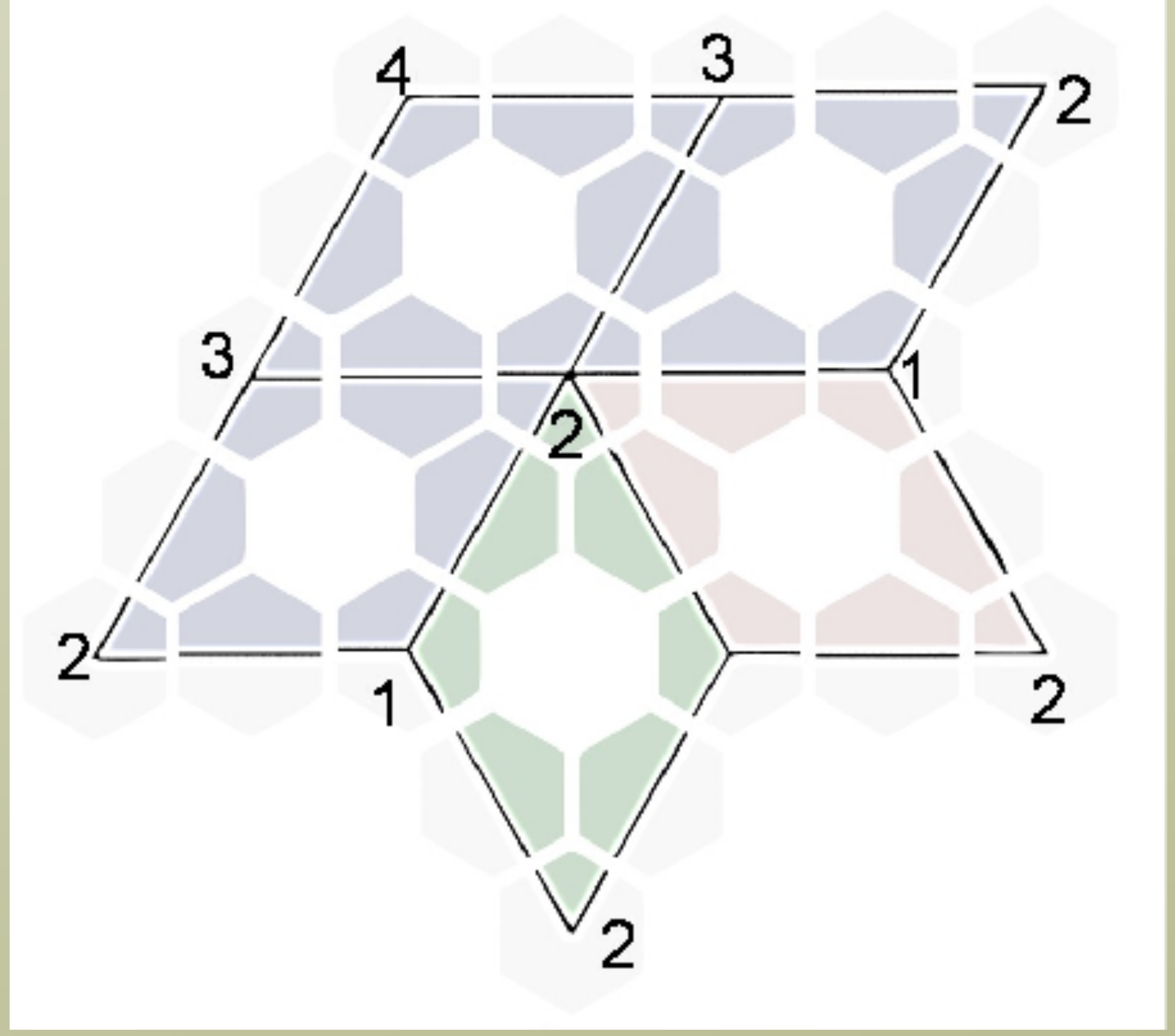
Infinite m-DIAMONDs (2D)
HCP (single layer)
b
c
a
[111]
Infinite m-CUBEs (2D)
HCP (tri layers)
[111]
b
Finite, r-CUBEs (2D)
Supramolecular particle (tetrahedron)





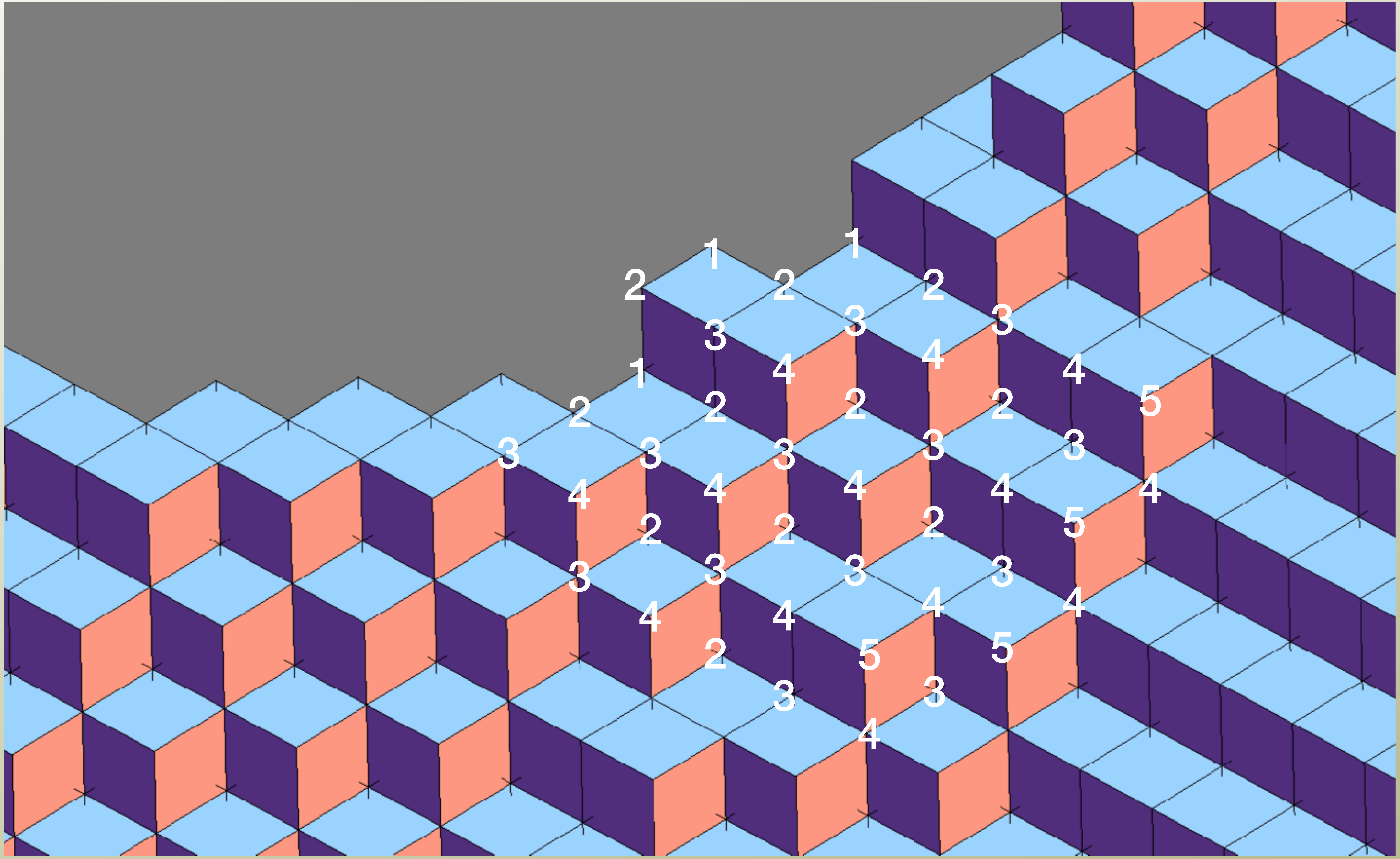




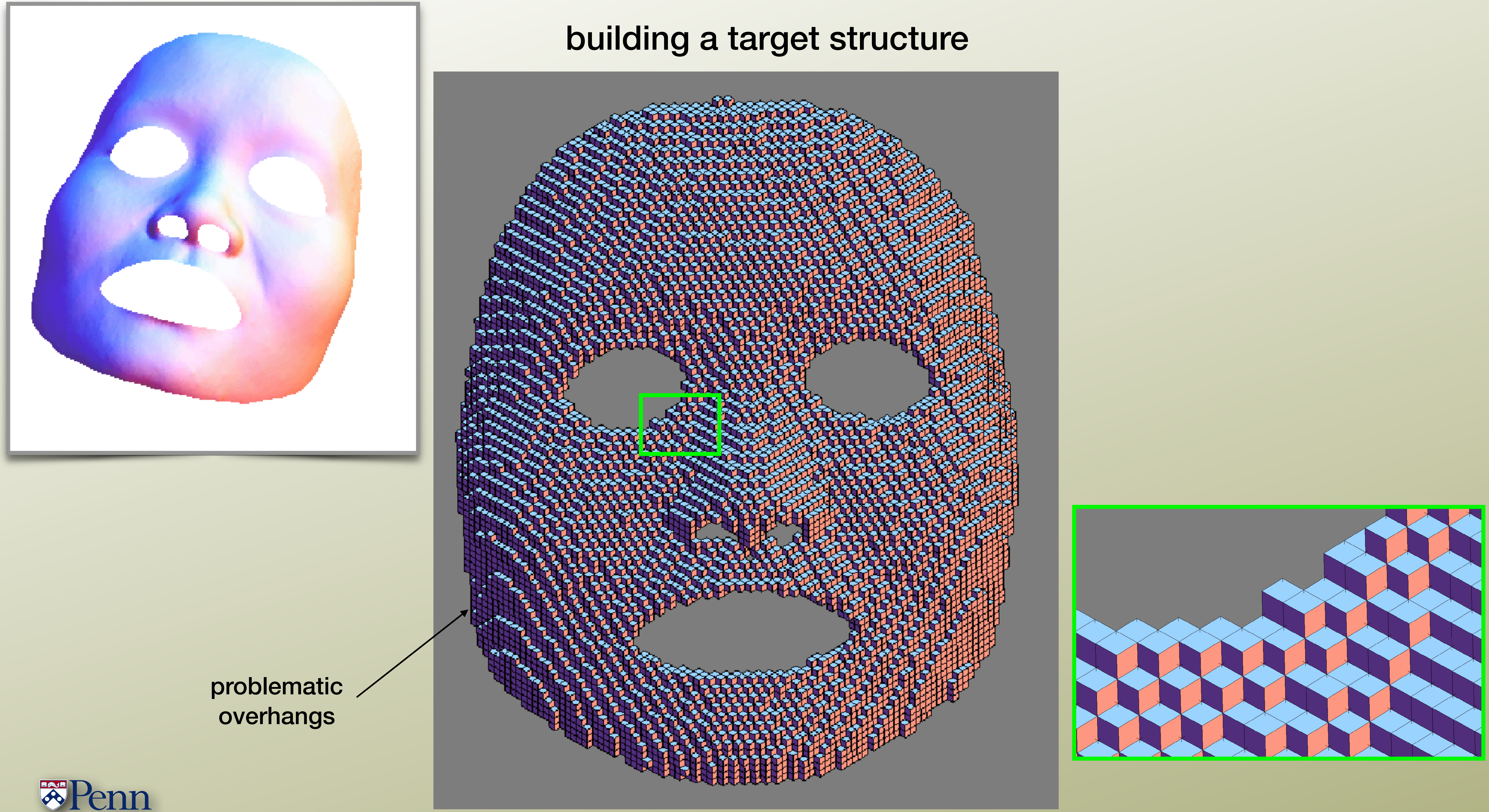


this time look at the numbers

we can interpret our height-labelled diamond packing as a non-overhanging cube packing viewed from the 111 direction



building a target structure



Thanks