Master internship proposal: Correct rounding of the `atan2` function

Advisor. Paul Zimmermann (INRIA Nancy - Grand Est, LORIA), Nancy, France.

Context. The IEEE 754 standard defines *correct rounding*: given a mathematical function f and a floating-point number x, the correct rounding of $f(x)$ is the floating-point number y closest to $f(x)$ according to the given rounding mode (to nearest, towards zero, towards $-\infty$ or towards $+\infty$). While IEEE 754 *requires* correct rounding for the basic arithmetic operations (addition, subtraction, multiplication, division, square root), it only *recommends* correct rounding for mathematical functions (exp, sin, pow, ...) The current mathematical libraries do not guarantee correct rounding [2]. The CORE-MATH project [1] aims at providing correctly rounded routines, for integration in these mathematical libraries.

Internship objectives. The goal of this internship is to design an efficient algorithm to compute the correct rounding of $\mathrm{atan}(y/x)$ (function `atan2` from C99, in double precision), and to efficiently implement this algorithm in the C language. Since `atan2` is not algebraic, one of the difficulties will be to obtain a bound on the maximal number of consecutive zeros or ones after the rounding bit. This corresponds to the *worst cases* and is known as the "Table Maker Dilemma" [3]. A possible research direction is to try to extend the approach described in [4] for the `atan2f` function in single precision. The implementation in the C language will be done within the CORE-MATH project, to make integration possible in the current mathematical libraries (GNU libc, Intel Math Library, AMD LibM, ...)

Prerequisites. This internship requires a solid mathematical background, and a good knowledge of the C language, especially to write efficient code.

References

[1] The CORE-MATH project. `https://core-math.gitlabpages.inria.fr/`.

[2] Innocente, V., and Zimmermann, P. Accuracy of mathematical functions in single, double, extended double and quadruple precision. `https://members.loria.fr/PZimmermann/papers/accuracy.pdf`, 2022. Version of August 29, 21 pages.

[3] Lang, T., and Muller, J.-M. Bounds on runs of zeros and ones for algebraic functions. In *Proceedings of the 15th IEEE Symposium on Computer Arithmetic* (2001), IEEE Computer Society, pp. 13–20.

[4] Sibidanov, A., Zimmermann, P., and Glondu, S. The CORE-MATH Project. In *ARITH 2022 - 29th IEEE Symposium on Computer Arithmetic* (virtual, France, Sept. 2022). `https://hal.inria.fr/hal-03721525`.